PGIM
MULTI-ASSET SOLUTIONS

PRIVATE *vs.* PUBLIC INVESTMENT STRATEGIES

Reported and Real-World Performance

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Reported **performance for a private asset class (*e.g.*, private equity buyout funds) assumes an investor's allocation is always fully invested in a highly diversified pool of private assets (*e.g.*, funds). However, there are many *real-world* constraints that inhibit a CIO from experiencing this reported performance.**

In practice, a CIO must follow an investment strategy to achieve a portfolio allocation to private assets. Such a strategy involves investing in only a subset of funds currently available (not the universe of funds), following a particular commitment pacing strategy, and temporarily holding uncalled and uncommitted capital in another asset class (say, a public market index or cash). Fund-selection uncertainty, commitment pacing, and the uncalled and uncommitted components are important contributors to a private investment strategy's real-world performance. Consequently, a CIO's private asset investment strategy is unlikely to experience the reported asset class performance.

In contrast, reported performance for a public asset class (*e.g.*, publicly listed equities) closely matches its real-world performance since a CIO can, if desired, immediately and fully invest in the entire asset class. In other words, the performances of an allocation to public assets and of its associated public asset investment strategy are identical.

When making asset allocation decisions, CIOs often consult historical reported asset class performance. However, comparing private and public asset class reported performance can be misleading as it does not incorporate the constraints of achieving a private asset portfolio allocation. Instead, comparing private and public real-world asset class performance must be done at the investment strategy level.

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Our Fair Comparison (FC) framework is a methodology to produce real-world performance measures for an investment strategy. With the FC framework, CIOs can compare private and public investment strategies on a consistent, risk-adjusted basis and make better-informed asset allocation decisions.

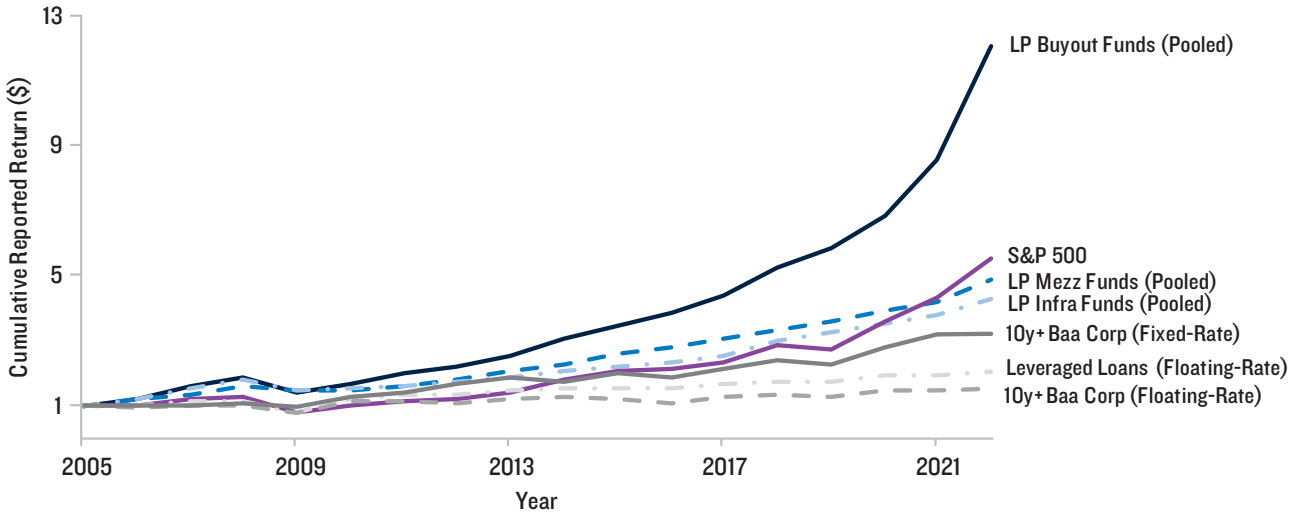
For 2005-2021, we find that real-world means and volatilities for private investment strategies are significantly different from their reported values. Specifically, using real-world returns we find that a buyout investment strategy outperformed mezzanine and infrastructure investment strategies, which is not apparent from reported returns. Also, a strategy of investing in public 10y+, fixed-rate Baa-corporate bonds has been reasonably competitive with private investment strategies.

Reported Performance

To illustrate the utility of the FC framework, first consider the **traditional comparison** of private and public asset performance that uses reported annual return data: pooled IRRs for private assets and index returns for public assets (see Appendix 1 for data details). Figure 1 shows the cumulative reported annual returns of a $1 investment for various private and public assets from 2005 to 2021. Private equity buyout funds outperformed public assets by a wide margin. However, as we will show, this traditional comparison is misleading because reported returns of private assets do not reflect the real-world performance that CIOs experience.

Figure 1: Traditional Comparison of Private *vs.* Public Asset Reported Performance

Cumulative Reported Annual Returns; 2005-2021



Note: Private asset annual returns are 1y IRRs computed by pooling cash flows and NAVs across all funds from all vintages for US buyout funds (minimum $250m capitalization), US mezzanine funds, and global infrastructure funds, respectively. Cumulative returns are compounded annual returns over the 2005-2021 period, assuming a $1 initial investment. Source: Bloomberg, Burgiss, S&P and PMA. Provided for illustrative purposes only.

To make a portfolio allocation to a private or public asset class a CIO must follow an **investment strategy**. In other words, after deciding on an amount (or portfolio percentage) to allocate to an asset class, the CIO must then execute an investment strategy to achieve this allocation. Depending on the asset class, such an investment strategy can add a performance drag or boost beyond the reported asset class performance. It is the performance of the investment strategy that is the "real-world" performance of the asset class.

The real-world performance of an asset class (public or private) measures the performance of an investment strategy that a CIO needs to execute to achieve a portfolio allocation to the asset class.

To achieve an intended investment amount in a public asset class, a CIO's investment strategy is relatively simple. The CIO can invest the allocation immediately – and start earning the reported asset class return – *via* a public market index, with any subsequent distributions immediately reinvested. Consequently, reported and real-world public asset class performances are the same.

The real-world performance of a public asset investment strategy will typically be the same as the asset's reported performance.

In contrast, to make a portfolio allocation to a private asset class generally requires a more challenging investment strategy. A CIO's private investment strategy allows investment in only a subset of available funds, not in the asset class as a whole – introducing both an element of idiosyncratic fund-selection risk & return in the investment strategy and a need to pace commitments to achieve fund vintage diversity that underlies the asset class's reported returns. Once the CIO selects and commits the intended investment amount to a GP fund(s), the CIO must wait for the GP to call the commitment(s) – usually in parts, spread over time. Meanwhile, the CIO invests any uncalled committed amounts, plus any yet-to-be committed amounts, in a public market index or cash. Consequently, reported private asset class performance measures (*e.g.*, pooled IRRs) – which assume an investor's allocation is always 100% invested in a highly diversified set of funds – are misleading because they do not incorporate fund selection, commitment pacing and the performance of uncalled and uncommitted capital, which are all an integral part of a private investment strategy's real-world performance.

To achieve an allocation to a private asset class, a CIO's investment strategy will involve fund-selection uncertainty, commitment pacing, and the investment return on any uncalled and uncommitted capital. Consequently, the CIO's real-world performance will not match the asset class's reported performance.

CIOs must decide how to allocate between public and private investments. The first step in this decision-making process is usually an analysis of historical performance. Our FC framework is designed to measure performance that incorporates the real-world challenges and constraints involved with private investment strategies including private equity, private credit, and infrastructure LP fund investments (excluding hedge funds). CIOs can use the FC framework to fairly compare the historical performance of illiquid private and liquid public investment strategies.

Real-World Performance – Fair Comparison

Figure 2 summarizes the FC framework (see Appendix 2 for details). The first step of the framework is to construct a private investment strategy, represented by a **composite private investment portfolio** (Figure 2, Step 1). We assume investors start by allocating $1 to the strategy and make new commitments following a simple rule: commit, say, 50% of any uncommitted capital at the beginning of each vintage year, equally divided between two funds (irrespective of fund size) randomly selected from the vintage, and then wait for the GP's capital calls.[1] Any committed, but uncalled, capital and any uncommitted capital is invested in a default public market index (*e.g.*, the S&P 500 Index). All capital calls are financed by liquidating a portion of the default investment and all distributions received are held as uncommitted capital and absorbed back into the default index investment. Total assets always stay invested, either in the private investment as NAV (*i.e.,* "in the ground") or in the default index investment. No extra capital flows in or out of this composite portfolio, thus making it *self-contained* and *self-financed*.[2] The FC framework is flexible and can accommodate other assumptions to better match investors' particular private investment strategies (*e.g.,* the number of funds selected each year and different commitment pacing strategies).

The second step in the FC framework is to generate a **terminal wealth distribution** for the private investment strategy. We rely on terminal wealth due to its reliability and relevance: first, terminal wealth measures the realized return based on *actual distributions* as well as *reinvestment of the distributions*, which are less sensitive to periodic GP valuations that can be subjective; second, since investors know their capital is inaccessible until returned by the GP, they are concerned with money received at the end of the investment, which is what terminal wealth measures.

As the hypothetical terminal wealth distribution shows (Figure 2, Step 2), the FC framework assumes investors start with allocating $1 to the private investment strategy and follows the strategy until the end of the investment horizon (*e.g.,* 10y). The framework applies the TA model to project cash flows for the specific two funds selected in each vintage year and calculates horizon terminal wealth.[3] The terminal wealth has a range of outcomes, depending on two sources of variation: 1) which two funds are selected in each vintage year, and 2) what returns are realized by both the funds selected and the default public index each year. Our framework

1 For simplicity, we assume LPs have no fund-selection skill. See, *Measuring the Value of LP Fund-Selection Skill: A Fair Comparison Framework* (PMA, April 2020), that measures the impact of LP fund-selection skill on private investment strategy performance. Also, we assume (unrealistically) that the LP has access to all funds in a given vintage year.

2 We assume the self-contained, self-financed portfolio is long-only. In practice, such a portfolio can face liquidity issues, such as not having enough liquid capital to meet capital calls. This is not a material concern for our analysis which assumes 50% of uncommitted capital is committed, and the remaining 50% of uncommitted capital is likely more than sufficient to meet capital calls. Any loss of capital will only reduce the size of future commitments.

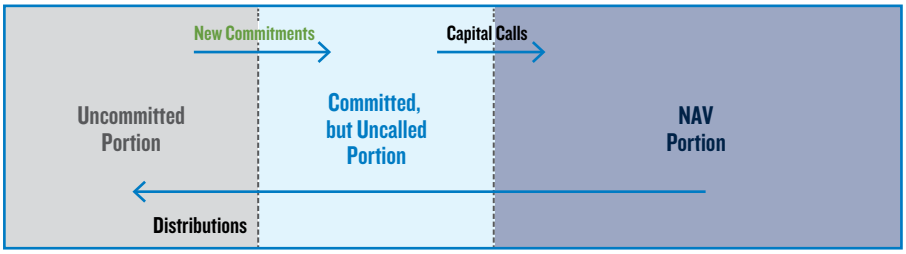
3 The TA model refers to the Takahashi and Alexander (2002) cash flow model that captures the stylized pattern of LP capital contributions, distributions, and NAVs. Appendix 2 contains details on the TA model and parameter calibration.

Figure 2: Summary of the FC Framework

STEP 1
Construct a Private Investment Strategy

A Composite Investment Portfolio with $1 Initially Invested

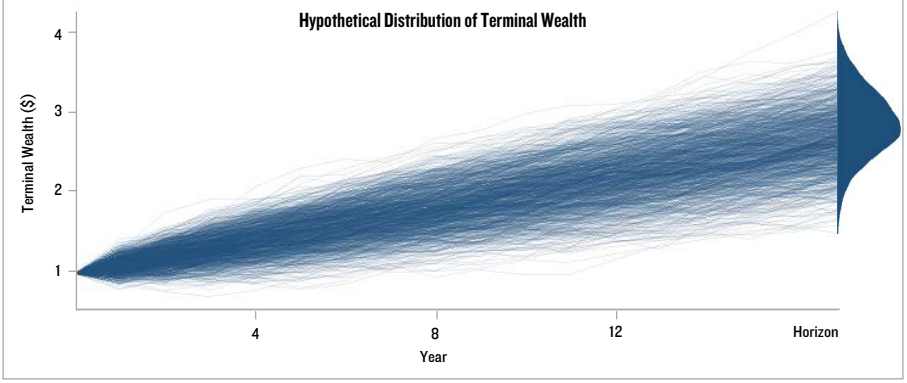
Commit 50% to 2 New Private Funds in Each Vintage Year



Invested in **Public Index** (*e.g.*, S&P 500) | Invested in **Private Funds**

STEP 2
Generate Terminal Wealth Distribution for the Private Investment Strategy

- Terminal wealth path varies, depending on 1) which funds are selected in each vintage year, and 2) what returns are realized by each fund selected and by the public index, respectively, in each year



STEP 3
Estimate Mean and Volatility of the Private Investment Strategy

- Infer the mean and volatility that would generate the terminal wealth distribution

STEP 4
Compare Private *vs.* Public Investment Strategy Performance

- Calculate mean/volatility ratio to adjust for risk
- Add leverage to bring the risks of all strategies to the same level
- Add financing costs incurred by leverage, as well as manager alpha and fees

Note: Key assumptions are underlined. Source: PMA. Provided for illustrative purposes only.

uses simulation to see how terminal wealth can vary depending on the funds selected, fund performance, and default public index performance.[4]

The third step in the FC framework is to work backwards from the terminal wealth distribution and estimate the real-world mean and volatility of the private investment strategy consistent with this terminal wealth distribution.

We also generate a terminal wealth distribution for public investment strategies. However, as a public investment strategy's real-world performance is the same as its reported performance, the mean and volatility of a public investment strategy's terminal wealth distribution correspond directly to their reported values.

The fourth, and final, step in the FC framework is to compare risk-adjusted returns (*i.e.*, mean/volatility ratios) of private and public investment strategies. To make volatilities comparable, the framework adds leverage to bring the risks of all strategies up to the same absolute level (to match the volatility of the highest volatility strategy), and accounts for financing costs incurred by leverage (interest, financing spread and haircut).[5] Finally, the framework adds manager alpha and fees to public strategy returns as private strategy returns are active returns, net of fees.

We follow a given private investment strategy for 17 years beginning with $1 in 2005 and ending in 2021. Each terminal wealth outcome depends on the specific funds selected by the strategy as well as the returns of each fund selected and of the default public index. We then simulate 10,000 possible strategies, capturing variations in fund-selection as well as in returns realized in each period, and generate a terminal wealth *distribution* of the strategy's outcomes. Figure 3 shows the terminal wealth distributions for buyout, mezzanine, and infrastructure investment strategy outcomes at the end of 2021.[6] From these distributions we work backwards and infer a strategy's mean and volatility consistent with its distribution. These values are the strategy's real-world mean and volatility.

Figure 4 compares the real-world *vs.* reported means and volatilities of private investment strategy returns. The real-world mean reflects the performance of investing in a subset of private funds (the NAV portion) as well as in the default index (the uncalled and uncommitted portions), and so differs from the reported mean which reflects only the average pooled performance of all funds (the NAV portion only). The real-world volatility captures the uncertainty from fund selection as well as the returns of each fund selected (the NAV portion) and the default public index (the uncalled and uncommitted portions). The real-world volatility is generally *higher* than the reported volatility.[7] The cross-sectional volatility for mezzanine funds, adjusted for mean, is higher than for buyout and infrastructure funds. Consequently, the difference in reported *vs.* real-world volatility for the mezzanine strategy (6.56%/y *vs.* 12.39%/y) is larger than for the buyout and infrastructure strategies.

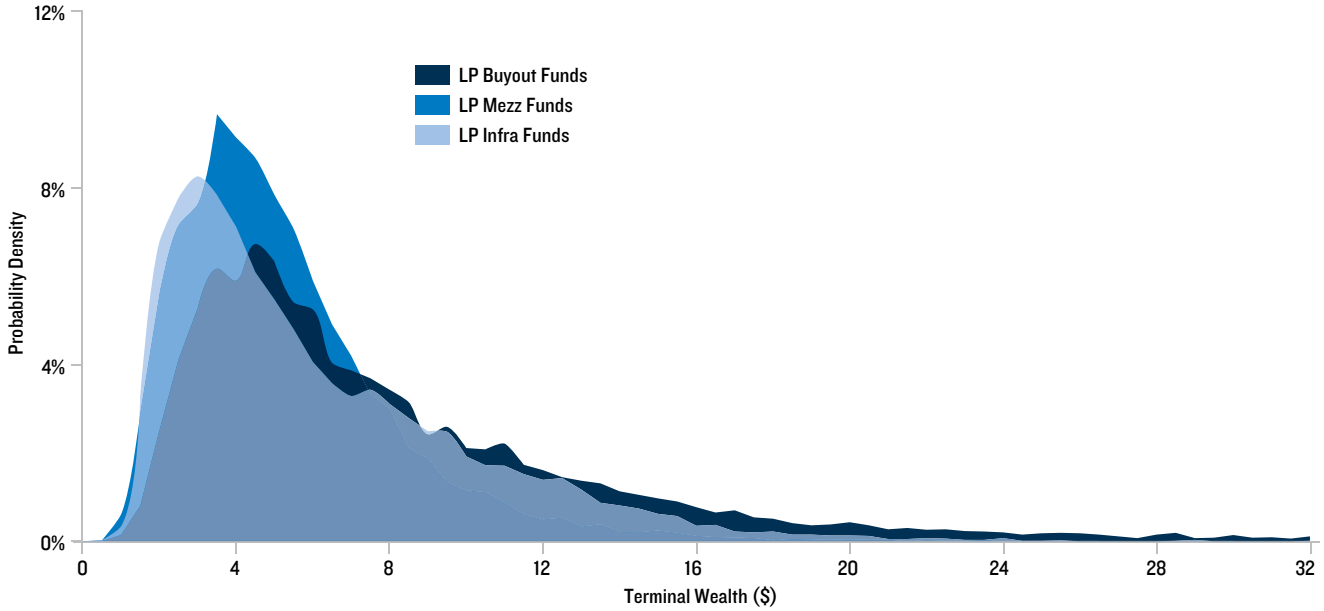
4 For each fund we use vintage-level TA model parameters for contributions, distributions, and NAVs, and fund-level 10y IRR as its growth parameter, assuming a 10y fund lifespan. To incorporate fund-level return variation, for each vintage, funds are sorted by their 10y IRRs and are evenly grouped into high/medium/low 10y IRR buckets (the best and worst IRRs are excluded to reduce potential outlier bias). For vintages with less than six funds, we group all funds into one bucket. For each bucket, we calculate the mean and volatility of 10y IRRs. We assume the default public index return is Normally distributed with its historical mean and volatility during 2005-2021, consistent with the investment horizon. Its distribution is evenly grouped into high/medium/low buckets, each with 1/3 probability, representing a good/medium/bad market scenario, respectively. For each year and in each market simulation run, we first randomly draw a default public index return and determine its bucket and market scenario. To link private fund returns to the default public index returns, we then randomly draw each fund's growth parameter from a Normal distribution with a mean and a volatility from the corresponding bucket as the default public index return.

5 Volatilities need to be comparable for the comparison to be fair since investment strategies with different volatilities have different portfolio implications and should not be considered to have the same portfolio role even if they have comparable mean/volatility ratios.

6 The buyout distribution has a longer right tail than mezzanine and infrastructure distributions as the best buyout funds perform better than the best mezzanine and infrastructure funds, resulting in larger terminal wealth. Some of the interim distributions are bimodal, potentially due to the private investment portfolios containing two processes – the pure private asset return process and the default public index return process – with different characteristics (*e.g.,* different means). An alternative approach to better capture the features of the terminal wealth distributions, when estimating the mean and volatility of private investment strategies, is a potential area of future research.

7 We assume the private investment strategies start in 2005 with zero existing investment and find the real-world means differ notably from reported ones. To evaluate the impact of starting with some existing investment amount, we estimate the means starting from years 6, 7, 8 (*i.e.*, 2010-2021, 2011-2021, 2012-2021), *etc.*, respectively. We find that the real-world means still diverge from reported ones without any clear evidence of convergence as they have different measurement scopes (*e.g.*, unlike reported means, real-world means incorporate the performance of uncalled and uncommitted capital).

Figure 3: Terminal Wealth Distribution of Private Investment Strategy Outcomes ($1 Initially Invested)



Source: Burgiss, S&P and PMA. For simplicity, a few larger than $32 terminal wealth outcomes are not shown. Provided for illustrative purposes only.

Figure 4: Reported and Real-World Performance; Private *vs.* Public Investment Strategies; 2005-2021

Investment Strategy	Asset	Reported Performance			Real-World Performance		
		Mean (%)	Vol (%)	Mean / Vol	Mean (%)	Vol (%)	Mean / Vol
LP Buyout Funds	Private	16.60	13.10	1.27	13.16	15.18	0.87
LP Mezz Funds	Private	9.99	6.56	1.52	9.93	12.39	0.80
LP Infra Funds	Private	9.44	10.26	0.92	10.99	14.43	0.76
S&P 500	Public	12.02	16.65	0.72	12.02	16.65	0.72
Leveraged Loans (Floating-Rate)	Public	5.28	14.83	0.36	5.28	14.83	0.36
10y+ Baa Corp (Floating-Rate)	Public	3.46	14.52	0.24	3.46	14.52	0.24
10y+ Baa Corp (Fixed-Rate)	Public	7.76	11.93	0.65	7.76	11.93	0.65

Note: All return numbers are annualized. The default public market index is assumed to be the S&P 500 Index. Reported performance is calculated using reported annual returns. The 2005-2021 pooled lifetime IRRs for buyout, mezzanine and infrastructure funds are 14.69%/y, 9.45%/y and 8.17%/y, respectively, which are weighted by fund capitalization. They differ from the reported means, which are calculated as an arithmetic mean of reported annual IRRs, where each IRR is equally weighted and the impact of year-over-year fund capitalization variation is ignored. Real-world volatility is computed as a 5-year moving average (*e.g.*, the volatility for horizon ending in 2021 is calculated as an average of the volatilities for horizons ending in 2017, 2018, 2019, 2020 and 2021, respectively) to reduce potential outlier bias (*e.g.*, volatility spike). Source: Bloomberg, Burgiss, S&P and PMA. Provided for illustrative purposes only.

Investment strategies with different risks have different portfolio implications and should not be considered to have the same portfolio role even if they have comparable mean/volatility ratios. To compare performance on an equal-risk basis, we assume investment strategies use leverage to bring their risks to the same risk level as that of the strategy with the highest *ex-post* volatility (*i.e.*, the S&P 500 Index in this case).[8] We then account for the financing costs incurred by any leverage, including financing spread and haircut, in addition to interest cost.

To complete the FC framework, we make sure that public and private investment strategy performances are consistent in their treatment of manager skill (or "alpha") and fees. As mentioned, private investment strategy returns reflect any value added provided by the GP. These returns are also net of fees. In contrast, public index returns exclude manager alpha and fees. Therefore, we add a reasonable level of *variable* manager alpha and *fixed percentage* fees to public investment strategy returns to be consistent with private investment strategy returns. Figure 5 presents the assumptions for leverage costs, manager alphas and fees, which can be customized to reflect investors' real-world situation (note: FC results are relatively insensitive to these assumptions).

8 We recognize that our *ex-post* analysis of levered returns is not replicable as it assumes the realized *ex-post* volatilities are known beforehand. Proper *ex-ante* analysis requires accurate prediction of both current and target volatilities. However, estimating volatility, which can vary over time, is difficult.

Figure 6 first compares levered returns for private and public investment strategies, but only considering the interest (3m LIBOR, or 3mL) expenses, without manager alpha, fees, financing spread & haircut. Levered returns can be viewed as a leverage-weighted average of unlevered returns and interest cost. Interest cost decreases levered means and increases levered volatilities, pushing mean/volatility ratios lower (Figure 4 *vs.* Figure 6; *e.g.*, the buyout mean/volatility ratio decreases from 0.87 to 0.83). As intended, Figure 6 shows that all investment strategies have similar volatilities.

The final FC real-world returns incorporate manager alpha, fees, financing spread and haircut (Figure 6). The financing spread and haircut dampen levered means (*e.g.*, buyout mean decreases from 13.81%/y to 13.58%/y). For public investment strategies, the variable manager alpha increases levered means and levered volatilities (*e.g.*, the S&P 500 mean increases from 12.02%/y to 12.51%/y and volatility increases from 16.65%/y to 16.69%/y), while the fees weaken levered means. The financing haircut reduces the effective level of leverage and lowers levered volatilities (*e.g.*, the buyout levered volatility decreases from 16.65%/y to 16.28%/y).[9]

Figure 5: Manager Alpha, Fees and Leverage Financing Cost – Assumptions

Investment Strategy	Manager Alpha (bp)	Manager Fees (bp)	Interest	Financing Spread (bp)	Haircut (%)
LP Buyout Funds	-	-	3mL	100	25
LP Mezz Funds	-	-	3mL	100	25
LP Infra Funds	-	-	3mL	100	25
S&P 500	50 (100)	15	-	-	-
Leveraged Loans (Floating-Rate)	100 (200)	45	3mL	100	25
10y+ Baa Corp (Floating-Rate)	50 (100)	15	3mL	50	10
10y+ Baa Corp (Fixed-Rate)	50 (100)	15	3mL	50	10

Note: All numbers are annualized. To generate variable manager alpha for public investment strategies, we sample Normally distributed alpha from the mean and standard deviation (in parenthesis) specified above.
Source: PMA. Provided for illustrative purposes only.

Figure 6: Real-World (Levered) & Fair Comparison Performances; Private *vs*. Public Investment Strategies; 2005-2021

Investment Strategy	Asset	Real-World (Levered) Performance			Fair Comparison Performance		
		Mean (%)	Vol (%)	Mean / Vol	Mean (%)	Vol (%)	Mean / Vol
LP Buyout Funds	Private	13.81	16.65	0.83	13.58	16.28	0.83
LP Mezz Funds	Private	11.12	16.69	0.67	10.56	15.61	0.68
LP Infra Funds	Private	11.68	16.66	0.70	11.39	16.10	0.71
S&P 500	Public	12.02	16.65	0.72	12.51	16.69	0.75
Leveraged Loans (Floating-Rate)	Public	5.74	16.69	0.34	6.19	16.58	0.37
10y+ Baa Corp (Floating-Rate)	Public	3.73	16.69	0.22	4.18	16.61	0.25
10y+ Baa Corp (Fixed-Rate)	Public	10.20	16.85	0.61	9.79	16.44	0.60

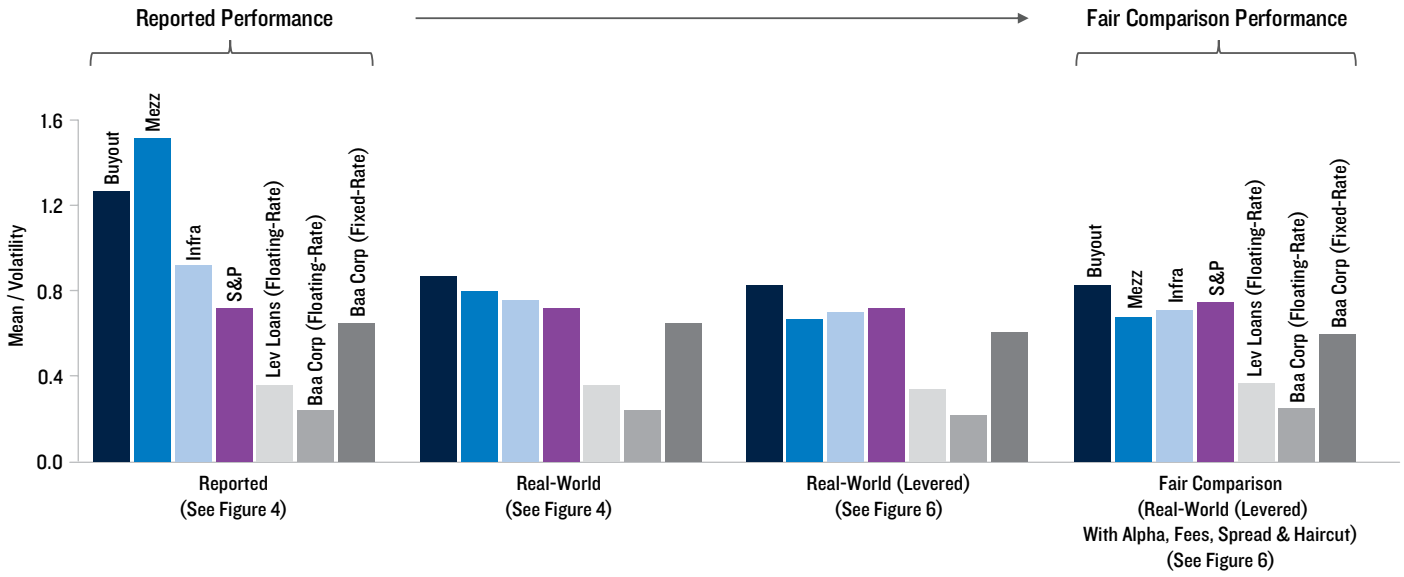
Note: All return numbers are annualized. The default public market index is assumed to be the S&P 500 Index. Alphas, fees and financing costs are based on the assumptions in Figure 5. Source: Bloomberg, Burgiss, S&P and PMA. Provided for illustrative purposes only.

To facilitate comparison, Figure 7 shows both reported and real-world performances across both private and public investment strategies. Figure 7 also shows the intermediate steps in the performance calculation before arriving at the final FC performance. For 2005-2021, real-world means and volatilities for private investment strategies are significantly different from their reported values. After using leverage to adjust risk to the same absolute level, buyout investment strategy outperformed mezzanine and infrastructure investment strategies, and a strategy of investing in public 10y+, fixed-rate Baa-corporate bonds has been reasonably competitive with private investment strategies.

9 Levered return: $R_{levered} = (R_{unlevered} + \alpha - fees) \times L_{haircut} - (interest + spread) \times (L_{haircut} - 1)$, *where* $L_{haircut} = 1 + (L-1) \times (1 - haircut)$ *and* $L = \sigma_{highest} / \sigma$.
Levered volatility: $\sigma_{levered} = \sqrt{(\sigma_{unlevered}{}^2 + \sigma_\alpha{}^2) \times L_{haircut}{}^2 + \sigma_{interest}{}^2 \times (L_{haircut} - 1)^2}$, assuming $R_{unlevered}$, α and interest are uncorrelated with each other.

We recognize that the 2005-2021 period is a low-interest rate era, and both traditional and fair comparison results can be sensitive to the choice of investment horizon.

Figure 7: Fair Comparison of Private *vs.* Public Investment Strategy Performance; 2005-2021



Note: All return numbers are annualized. The default public market index is assumed to be the S&P 500 Index. Source: Bloomberg, Burgiss, S&P and PMA. Provided for illustrative purposes only.

Fair Comparison Results – Impact of Assumptions

FC results are sensitive to assumptions, as different assumptions represent different investment strategies which can lead to different terminal wealth outcomes. For example, we have assumed that any uncommitted and committed, but uncalled, capital is invested in the S&P 500 Index. However, some investors may assume a default investment of cash. The FC framework is flexible and can accommodate various assumptions to reflect an investor's own real-world investment strategies. How sensitive are the FC results to these assumptions?

Using a buyout investment strategy as an example, we provide sensitivity analyses for various assumptions, including the default public market index (assumed to be the S&P 500 Index), the commitment pacing strategy (we assume 50% of any uncommitted capital at the beginning of each vintage year is committed), the number of funds selected (we assume two funds are randomly selected in each vintage), and the fund lifespan (assumed to be 10 years).[10]

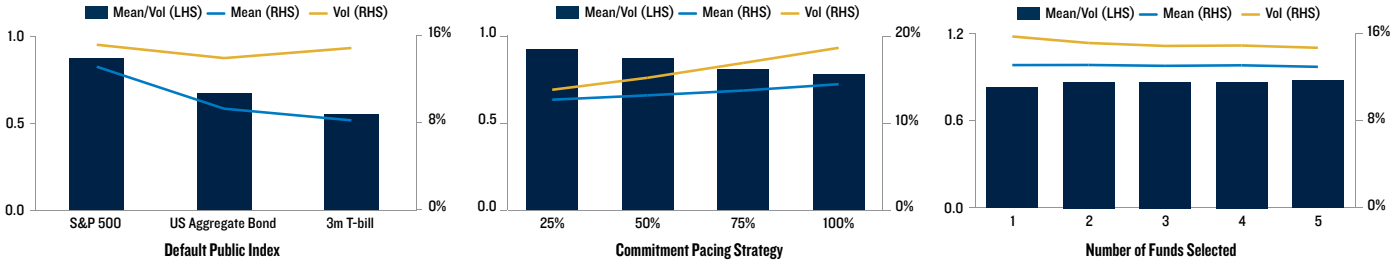
When the default public market index is the S&P 500 Index, which has higher risk and return relative to some alternatives we consider (US Aggregate Bond Index and 3m T-bill), the buyout investment strategy has a higher real-world mean, volatility, and a higher mean/volatility ratio (Figure 8, left).

The impact of faster commitment pacing on real-world performance is less clear-cut. On the one hand, a larger portion of uncommitted capital will be committed to private funds and more distributions will be received and absorbed into the default public index, resulting in higher terminal wealth. On the other hand, a smaller portion of uncommitted capital will stay in the default index reducing uncommitted capital and future commitments to private funds, leading to lower terminal wealth. The overall impact depends on the relative sizes of these two effects. In the scenarios we consider (commitment pacing of 25%, 50%, 75% and 100%), faster pacing makes terminal wealth larger and more dispersed, pushing both the real-world mean and volatility higher. However, the mean/volatility ratio declines (Figure 8, middle). Note that when 100% of uncommitted capital is committed to private funds, the real-world mean is 13.41%/y, lower than the reported 16.60%/y mean (Figure 4). This difference exists because the real-world mean includes the performance of uncalled capital invested in the default index as well as the NAV of a subset of funds, while the reported mean captures the average NAV performance of all funds. A cash flow matching commitment strategy, where all distributions received in the previous period fund all capital calls in the next period so that periodic net cash flows are close to zero, should bring the real-world mean somewhat closer to the reported mean.

10 Our base case assumptions are in parentheses. Each sensitivity analysis only changes one assumption and keeps all other assumptions unchanged. Number of simulations, once sufficiently large (say, 10,000), does not impact the mean/volatility ratio. For simplicity, the sensitivity analyses consider the real-world performance, ignoring leverage, financial spread, haircut, manager alpha and fees, the impact of which are relatively small.

An investment strategy in selecting and investing in more funds amplifies the diversification benefit – driving real-world volatility lower – and results in a higher mean/volatility ratio (Figure 8, right). However, selecting more funds (*e.g.*, selecting five funds each year leads to 85 funds by year 17) may not be reasonable as it would likely incur significant monitoring costs (assumed to be zero in our analysis).

Figure 8: Impact of FC Assumptions on Buyout Investment Strategy Performance



Note: All return numbers are annualized. Source: Barclays, Bloomberg, Burgiss, DataStream, S&P and PMA. Provided for illustrative purposes only.

Using the FC Framework – Portfolio Implications

To examine the portfolio implications of using real-world *vs.* reported performance for asset allocation, we consider an example of a portfolio comprising three investment strategies including buyout funds, 10y+ Baa Corp (Fixed-Rate) and 3m T-bill, with a 9%/y portfolio volatility target.

Figure 9 assumes that investors make portfolio allocation decisions based on a set of *ex-ante* assumptions of investment strategy performance.[11] Using the real-world performance of the buyout investment strategy, with a lower mean and a higher volatility than reported performance, investors would have faced a lower efficient frontier and allocated less to buyout funds (46% *vs.* 61%). Consequently, investors would have realized a lower portfolio volatility (9.0%/y *vs.* 10.2%/y), meeting the volatility target, and a higher Sharpe ratio (0.90 *vs.* 0.88), which are the benefits of using real-world performance. In contrast, if investors used reported buyout performance statistics, the result would have been a higher realized portfolio volatility, breaching the volatility target, and a lower realized Sharpe ratio.

CIOs can use the FC framework, which can be customized based on their assumptions, to estimate the real-world performance of their private investment strategies and make better-informed asset allocation decisions.

11 For portfolio construction purposes, we consider the unlevered real-world performance for both private and public investment strategies, and passive performance without alpha and fees for public investment strategies (Figure 4).

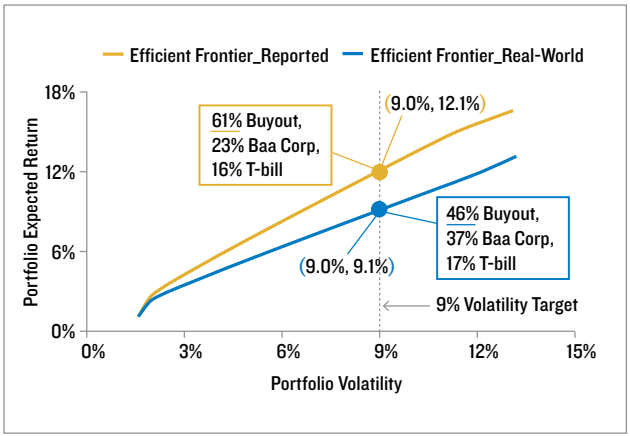
Figure 9: Impact of Using Real-World *vs.* Reported Performance on Portfolio Construction

Using Buyout – Baa Corp – T-bill Portfolio as an Example

1. *Ex-Ante* Assumptions

Asset	Real-World	Reported
Buyout Mean	13.2%/y	16.6%/y
Buyout Vol	15.2%/y	13.1%/y
Baa Corp Mean	7.8%/y	7.8%/y
Baa Corp Vol	11.9%/y	11.9%/y
T-bill Mean	1.1%/y	1.1%/y
T-bill Vol	1.6%/y	1.6%/y
Corr (Buyout, Baa Corp)	0.2	0.2
Corr (Buyout, T-bill)	0.3	0.3
Corr (Baa Corp, T-bill)	-0.2	-0.2

2. Asset Allocation Decision



3. *Ex-Post* Realized (*i.e.*, Real-World) Performance

Portfolio	Using Real-World Assumptions	Using Reported Assumptions
Mean	9.1%/y	10.0%/y
Vol	9.0%/y	10.2%/y
Sharpe Ratio	0.90	0.88

Note: Optimal portfolio construction involves many considerations that are not incorporated in this example. For example, liquidity risk considerations are absent. Real-world buyout correlations with Baa Corp and T-bill are unknown and assumed to be the same as reported correlations: 0.2 for buyout - Baa Corp and 0.3 for buyout - T-bill. Baa Corp - T-bill correlation is assumed to be -0.2, based on reported annual returns from 2005 to 2021. If real-world buyout correlations are higher than reported, the real-world efficient frontier will shift even lower, and the optimal buyout allocation will be lower as less buyout is needed to meet the same volatility target given higher correlation. The conclusions on reduced allocation to buyout, lower realized volatility and higher realized Sharpe ratio will still hold. The risk-free rate is assumed to be 1%/y. Source: Barclays, Bloomberg, Burgiss, S&P and PMA. For illustrative purposes only.

Appendix 1: Data

We compare the performance of investing $1 in private and public investment strategies using the following datasets:

Private Assets – LP US Buyout, US Mezzanine and Global Infrastructure Funds: Reported fund-level pooled annual IRR data are from Burgiss.[12] These data are LP-supplied, but GP-reported. These IRR data are also net of fees and include any manager active return (*i.e.,* skill or "alpha"). Buyout and mezzanine fund data are for the US market (same for public assets), while infrastructure fund data are for the global market as US data are limited. Burgiss database has about 1,400 US buyout funds, 280 US mezzanine funds, and 300 global infrastructure funds as of 2022 Q2. For US buyout funds, our analysis focuses on those with at least $250m capitalization, considering that institutional CIOs are unlikely to select smaller funds. For US mezzanine and global infrastructure funds where data are more limited (*e.g.*, less than 5 funds in some vintages), we consider all fund sizes to avoid further shrinking the sample sizes.

Public Assets: 1) **S&P 500 Index**, from S&P, measures the performance of the large-cap US equity market. 2) **Leveraged Loans (Floating-Rate)**, from S&P, is a broad coverage floating-rate index of broadly syndicated bank loans rated B or BB (at the beginning of each month), representing an investment in leveraged loans. The returns of all public indices are gross of fees and do not include manager active return or alpha. 3) **10y+ Baa x-Financials Corporate Index (Floating-Rate)**, from Bloomberg, is a broad coverage fixed-rate index of long duration, Baa-rated corporate (excluding financial companies) bonds. To remove performance related directly to Treasury duration, we convert periodic index returns to floating-rate using reported index excess returns and adding a 3m LIBOR return. 4) **10y+ Baa x-Financials Corporate Index (Fixed-Rate)** measures the total return of the same fixed-rate index. We compare this index with its constructed floating-rate version to allow identification of the Treasury duration component of returns over our data period, 2005-2021, a period of declining Treasury yields.

Appendix 2: The FC Framework Methodology

We present a FC Framework with *three major enhancements* over our prior study.[13] First, we apply a *new* private investment strategy terminal wealth generation algorithm using the TA model as the underlying engine to project cash flows for private funds selected by an investor's investment strategy. Second, we capture the strategy volatility in a *new* way by incorporating return variations in funds selected and in the default public index for a given year. Third, we develop a *new* methodology to estimate the real-world mean and volatility of private investment strategy returns from the terminal wealth distribution.

The first enhancement refines the process to generate terminal wealth. To measure private investors' ability to time and size their investments, we need detailed cash flow data by fund. With no access to such data, our previous approach generates synthetic cash flow data using funds' since-inception IRRs and TVPIs (Total Value to Paid-In). However, while extending the data from 2005-2018 to cover 2019-2021, we find the resulting cash flows do not match the typical patterns of private funds (*e.g.*, hump-shaped distributions over the investment horizon). Applying the TA model allows us to project future cash flows for private funds that better capture their actual behaviors, thus generating more reasonable terminal wealth.

The second enhancement is related to the volatility estimation. We incorporate the uncertainty in returns of each fund selected and of the default public index in each year, instead of only considering the uncertainty in fund selection, in the terminal wealth simulation process. From the terminal wealth distribution, we estimate the volatility which includes both sources of uncertainty.

The third enhancement addresses the issue of the Long (1999) method applied in our previous work.[14] After close examination and simulation, we find the Long (1999) method, as published, generates inaccurate estimates of real-world mean and volatility of private investment strategy returns. To produce better estimates, we present a new methodology using the mean of the terminal wealth distribution and the volatility of its log distribution.

Let TW denote terminal wealth with mean μ_{TW}, and r_i ($i = 1, \ldots, N$) denote unobservable *i.i.d.* returns for an investment strategy that has run for N vintage years, with mean μ_r and volatility σ_r.[15]

12 These are point-to-point IRRs (sometimes approximated using modified-Dietz return formula) computed by pooling cash flows and NAVs of all funds from all vintages.

13 *A Fair Comparison Framework: Risk and Reward in Private & Public Investments* (PMA, October 2019).

14 For details on the Long (1999) method, refer to Austin M. Long, "Inferring Periodic Variability of Private Market Returns as Measured by σ from the Range of Value (Wealth) Outcomes over Time," *The Journal of Private Equity 2, no. 4 (1999): 63–69.*

15 Assuming *i.i.d.* returns is reasonable for public markets with frequent trading and relatively full information. While this assumption does not hold in reported returns for private markets, which often require periodic estimated valuations that may change little over time, horizon terminal wealth is a known quantity. Using private investment strategy terminal wealth values to infer what could have been the parameters of the *i.i.d.* returns process if the market allowed frequent trading and relatively full information allows investors to better compare returns for public and private investment strategies.

TW is a function of compounded returns, starting with $1 invested:

$$TW = (1 + r_1) \times (1 + r_2) \times \ldots \times (1 + r_N) \tag{A1}$$

Taking the expected value of both sides gives:

$$\mu_{TW} = E(TW) = E[(1 + r_1) \times (1 + r_2) \times \ldots \times (1 + r_N)] \tag{A2}$$

Given the assumption that $r_i's$ are *i.i.d.*, we have:

$$\mu_{TW} = (1 + \mu_r)^N \tag{A3}$$

From (A3) we can express μ_r as:

$$\mu_r = \mu_{TW}^{1/N} - 1 \tag{A4}$$

To estimate σ_r, we take the log of both sides of (A1):

$$ln(TW) = ln(1 + r_1) + ln(1 + r_2) + \ldots + ln(1 + r_N) \tag{A5}$$

Given that $ln(1 + r_i) \approx r_i$, assuming r_i is small, we rewrite (A5) as:

$$\ln(TW) \approx r_1 + r_2 + \ldots + r_N \tag{A6}$$

Given that $r_i's$ are *i.i.d.*, we get:

$$\sigma^2_{\ln(TW)} \approx N \times \sigma_r^2 \tag{A7}$$

From (A7), σ_r can be expressed as a function of $\sigma_{\ln(TW)}$ and N:

$$\sigma_r \approx \sigma_{\ln(TW)} / \sqrt{N} \tag{A8}$$

Therefore, the estimates of μ_r and σ_r can be computed from Equations (A4) and (A8).

We recognize that (A8) gives an approximation of σ_r, as its accuracy rests on the assumption of r_i being small. This becomes an issue as the length of the period increases. For example, r_i will tend to get "large" (say, 10%) if we measure returns over an annual period. As the magnitude of r_i increases, (A8) tends to underestimate σ_r.

Long (1999) uses the upper and lower ranges of TW to infer μ_r and σ_r as follows:

$$TW_{max} = (1 + \mu_r^L)^N \times (1 + \sigma_r^L/\sqrt{N})^N \tag{A9}$$

$$TW_{min} = (1 + \mu_r^L)^N / (1 + \sigma_r^L / \sqrt{N})^N \tag{A10}$$

where TW_{max} and TW_{min} are the upper and lower ranges of TW, respectively. The superscript L denotes Long (1999) versions of μ_r and σ_r.

Long (1999) also sets a condition for TW_{max} and TW_{min}: they are within one standard deviation of μ_{TW}. We interpret this condition as (A11) and (A12):

$$TW_{max} = min\,[\mu_{TW} + \sigma_{TW}, TW_{max}] \tag{A11}$$

$$TW_{min} = max\,[\mu_{TW} - \sigma_{TW}, TW_{min}] \tag{A12}$$

Long (1999) then generates estimates for μ_r^L and σ_r^L as follows:[16]

$$\mu_r^L = (TW_{max} \times TW_{min})^{1/2-N} - 1 \tag{A13}$$

$$\sigma_r^L = [(TW_{max} / TW_{min})^{1/2-N} - 1] \times \sqrt{N} \tag{A14}$$

We use simulation to evaluate the new approach *vs.* Long (1999). We assume *i.i.d.* returns with $\mu_r = 10\%/y$ and $\sigma_r = 15\%/y$. We measure returns in years and examine various horizons (N from 5y to 40y) and run 100,000 simulations for each horizon.[17] To implement the simulation we must specify a distribution type (*e.g.,* Normal, uniform). We choose the Normal distribution which is a reasonable choice relative to other possibilities.[18]

Figure A1 shows the simulation results for the two approaches. The new mean estimate is 10%/y, identical to the actual mean (col. 6), and the new volatility estimate is 14%/y, lower than the actual volatility of 15%/y (col. 7). This is expected as the annual mean is relatively large. If we were to use a shorter period with a smaller mean, the new volatility estimate would be closer to the actual.[19] The new mean and volatility estimates are stable across various horizon lengths because (A4) and (A8) always hold. The estimates produce a stable 0.7 Sharpe ratio, close to the 0.67 actual value (col. 8).

16 Multiplying (A9) and (A10) gives $TW_{max} \times TW_{min} = (1 + \mu_r)^{2N}$, from which Long (1999) derives (A13). Similarly, dividing (A9) by (A10) gives $TW_{max} / TW_{min} = (1 + \sigma_r/\sqrt{N})^{2N}$, from which Long (1999) derives (A14). Note that, although not mentioned in Long (1999), $TW_{max} \times TW_{min}$ and TW_{max} / TW_{min} need to be positive for (A13) and (A14) to be valid.

17 Private market data back to 1990 are now widely available, so 30y or 40y horizons are feasible.

18 Our analysis only assumes *i.i.d.* returns, not the distribution type. Therefore, a different distribution type does not change the relative inaccuracy of the Long (1999) approach.

19 With quarterly returns, assuming $\mu_r = 2.4\%/q$ (10%/y when annualized) and $\sigma_r = 7.5\%/q$ (15%/y when annualized), the new volatility estimate would be 7.4%/q, closer to the actual.

In contrast, Long's (1999) estimates widely miss the mark (cols. 9 & 10). Estimates of the mean range from 5%/y to 9%/y, considerably lower than the actual 10%/y. As the horizon lengthens, the volatility estimate increases rapidly (*e.g.,* 25%/y for a 30y horizon), much higher than the actual 15%/y. Consequently, the Long (1999) Sharpe ratio estimates are too low and continue to fall as the horizon lengthens (col. 11). Figure A2 shows the Long (1999) estimates as a function of the horizon length.

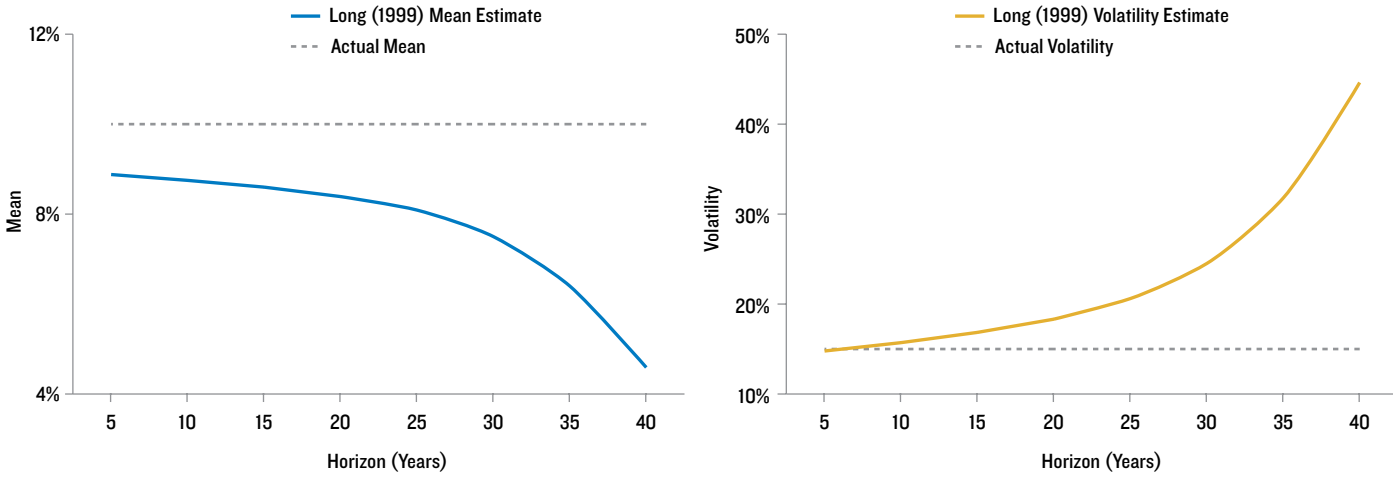
Figure A1: Mean and Volatility Estimates: New *vs.* Long (1999)

Row / Column No.	(1) Horizon (Years)	(2) TW Mean	(3) ln(TW) Vol	(4) TW Max	(5) TW Min	(6) New Mean Estimate	(7) New Vol Estimate	(8) New Sharpe Ratio Estimate	(9) Long (1999) Mean Estimate	(10) Long (1999) Vol Estimate	(11) Long (1999) Sharpe Ratio Estimate
(1)	5	1.6	0.3	2.1	1.1	10%	14%	0.7	9%	15%	0.6
(2)	10	2.6	0.4	3.8	1.4	10%	14%	0.7	9%	16%	0.6
(3)	15	4.2	0.5	6.5	1.8	10%	14%	0.7	9%	17%	0.5
(4)	20	6.7	0.6	11.2	2.2	10%	14%	0.7	8%	18%	0.5
(5)	25	10.9	0.7	19.2	2.5	10%	14%	0.7	8%	21%	0.4
(6)	30	17.5	0.8	32.6	2.4	10%	14%	0.7	7%	25%	0.3
(7)	35	28.1	0.8	54.8	1.4	10%	14%	0.7	6%	32%	0.2
(8)	40	45.3	0.9	92.1	0.4	10%	14%	0.7	5%	45%	0.1

Note: For simplicity the Sharpe ratio assumes a risk-free rate of 0%. Source: PMA. Provided for illustrative purposes only.

Figure A2: Long (1999) Mean and Volatility Estimates *vs.* Actual

(100,000 simulations of *i.i.d.* Normal returns with $\mu_r = 10\%/y$ and $\sigma_r = 15\%/y$)



Source: PMA. For illustrative purposes only.

Investors may have a view that private investment strategy returns truly exhibit some autocorrelated behavior. For example, new information on the likelihood of a private investment strategy's ultimate success or failure may be revealed slowly as uncertainty is gradually resolved. Consequently, an investor may be comfortable comparing public *i.i.d.* returns with private autocorrelated returns. We compare our new approach against Long (1999) assuming private investment strategy returns are indeed autocorrelated. We find that the new approach, although gives inaccurate estimates, still outperforms Long (1999). Details are available upon request.

Appendix 3: TA Model Parameter Calibration

We use the TA model to estimate NAVs and cash flows (*i.e.*, contributions and distributions) for each private fund selected by investors, based on the following equations.

Contribution Model:

$$C_t = UC_{t-1} \times RC(Age_{t-1}) \tag{A15}$$

where:

- UC_{t-1} is uncalled capital amount at the end of the last period
- Age_{t-1} is the number of years since the first capital call
- RC (rate of contribution) is a piecewise constant function of age of commitment

Distribution Model:

$$D_t = NAV_{t-1} \times (1 + G) \times \mathrm{RD}(Age_{t-1}, bow, L) \tag{A16}$$

$$\mathrm{RD} = max\,[Yield, (\,Age_{t-1} / L)^{bow}] \tag{A17}$$

where:

- D_t is distribution at the end of the period
- NAV_{t-1} is NAV at the end of the last period
- G is the expected growth rate
- bow controls the rate at which the distribution rate changes over time
- L is the expected lifespan
- RD (rate of distribution) is a function of age, bow and expected lifespan
- $Yield$ sets a minimum RD for income generating assets (*e.g.*, real estate) and is set to zero for other asset types

NAV Model:

$$NAV_t = NAV_{t-1} \times (1 + G) + C_t - D_t \tag{A18}$$

where:

- C_t is contribution at the end of the period

Figure A3 shows the calibrated TA parameters for historical vintage funds. US mezzanine funds have distributions that tend to ramp up earlier, thus having lower bow estimates than US buyout and global infrastructure funds. US buyout funds tend to experience slower capital calls, resulting in lower rate of contribution estimates than US mezzanine and global infrastructure funds. For historical vintage commitments, growth is historical reported lifespan IRR.[20] US buyout funds have higher valuations and hence higher growth than US mezzanine and global infrastructure funds.

20 For historical vintages with more than 15y of data, we use 15y reported IRRs. For historical vintages with more than 10y but less than 15y of data, we estimate 15y IRRs from 10y IRRs based on their historical linear relationship. For historical vintages with more than 5y but less than 10y of data, we estimate 15y IRRs from 5y IRRs based on their historical linear relationship.

Figure A3: Calibrated TA Parameters for Historical US Buyout, US Mezzanine and Global Infrastructure Funds

Vintage	Bow			Rate of Contribution (RC)									Growth		
	Buyout	Mezz	Infra	<1y			1-2y			>2y			Buyout	Mezz	Infra
				Buyout	Mezz	Infra	Buyout	Mezz	Infra	Buyout	Mezz	Infra			
2006	4.6	2.1	5.1	8%	10%	18%	11%	10%	12%	10%	14%	11%	8%	7%	2%
2007	5.1	2.3	4.0	9%	16%	21%	5%	7%	10%	7%	7%	10%	11%	10%	4%
2008	4.6	2.4	3.8	5%	12%	19%	3%	4%	7%	8%	11%	10%	15%	9%	9%
2009	3.7	2.7	3.6	3%	10%	6%	6%	4%	3%	9%	7%	10%	19%	8%	7%
2010	4.3	2.5	4.7	6%	11%	10%	5%	9%	4%	9%	11%	7%	14%	11%	7%
2011	4.3	2.5	3.9	7%	12%	12%	5%	4%	3%	8%	7%	9%	17%	10%	7%
2012	3.8	3.0	3.8	7%	10%	7%	4%	3%	4%	10%	11%	9%	19%	8%	12%
2013	3.8	1.7	3.5	3%	11%	8%	5%	8%	11%	8%	12%	17%	17%	10%	9%
2014	3.3	1.9	3.0	7%	6%	5%	6%	5%	2%	10%	11%	9%	17%	10%	11%
2015	2.9	2.6	2.7	6%	8%	9%	6%	6%	12%	11%	12%	16%	17%	10%	11%
2016	2.6	2.1	2.9	7%	5%	8%	8%	7%	6%	15%	10%	20%	17%	10%	9%
2017	2.3	2.0	3.1	6%	11%	17%	10%	12%	13%	16%	16%	18%	22%	10%	9%

Source: PMA. For illustrative purposes only.

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