PGIM QUANTITATIVE SOLUTIONS

# BUILDING AND EVALUATING BETTER RISK-AWARE STRATEGIC PORTFOLIOS

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## Executive Summary


The vast majority of investment risk and returns for institutional asset owners will depend on strategic portfolio allocation decisions. This raises the important challenge of how best to construct and evaluate different strategic allocations, keeping in mind that risk can be defined by more than just standard deviation. This challenge is exacerbated by the fact that many asset classes, and the multi-asset portfolios constructed with those assets, exhibit non-normal return distributions that violate the Gaussian assumptions inherent in approaches such as Mean-Variance Optimization (MVO) and standard multivariate normal simulation methods. To consider these issues, we evaluate a range of available portfolio construction options, considering different risk metrics. To better assess potential return paths for different allocations we also conduct forward-looking simulations that incorporate favorable and unfavorable investment regimes that are historically more representative of negatively skewed asset classes contained in multi-asset portfolios.

# I. Introduction

Strategic portfolio allocation is the most important investment decision asset owners need to make in order to achieve desired return targets and fund liabilities. Even with significant allocations to active managers or an active tactical asset allocation approach, the primary determinant of portfolio outcomes will be strategic allocations to underlying asset classes. A common approach to constructing strategic portfolios is through Mean-Variance Optimization (MVO), developed by Markowitz (1952), in which a frontier of feasible portfolios is constructed with expected returns and a covariance matrix of considered assets as inputs. Typically, ex-ante constraints are placed on individual asset classes or broad asset class groups to maintain a desired level of diversification. Of primary importance then, beyond initial constraints, is a robust methodology for the forecasting of long-run future returns and the covariance matrix. Beyond the challenging task of forecasting average time-varying return and risk parameters for an uncertain future, the MVO portfolio construction model assumes elliptical multivariate normal distribution for asset class returns, an assumption that is soundly at odds with actual asset class behavior.

The shortcomings of portfolio construction methods, principally MVO, incorporating the assumption that asset returns are normally distributed, have been well documented in the academic literature. Multiple studies have identified significant negative skewness and excess kurtosis in equity returns. Likewise, studies have observed that investors are averse to negative skewness and will seek hedging assets in longer horizon portfolios on the expectation of episodic periods of adverse investment regimes.

Merton (1973) introduces an intertemporal capital asset pricing model (ICAPM) in which an asset's expected return depends on its covariance with the market portfolio and with state variables that proxy for changes in the investment opportunity set. An insight of the ICAPM is that investors have hedging demands in the optimal portfolio to protect against stochastic shifts in the opportunity set of asset classes. In selecting their portfolios, investors consider events that they expect to occur beyond the current period. Specifically, investors take into account not just current-period returns and correlations, but also those they expect in future periods in an effort to hedge and maximize their portfolio returns. Barberis (2000) develops this concept further by empirically estimating how horizon and parameter uncertainty interact to determine the desired equity allocation.

Subsequent empirical studies highlight the importance of higher-order moments in historical asset returns, including negative skewness and excess kurtosis on realized portfolio outcomes. Harvey and Siddique (2000) define co-skewness as "the component of an asset's skewness related to the market portfolio's skewness." They analyze the ability of co-skewness to explain the cross-sectional variation of equity returns in comparison with other factors and find that systematic negative skewness is economically important and commands a risk premium, on average, of 3.60% per year. Given the importance of this typically unincorporated risk in the mean-variance optimization problem, they recommend that "a richer conditional mean-variance-skewness framework may be employed" to better represent investor utility. Engle (2004) highlights the effect of asymmetry in variance for multi-period returns. For example, even though each period has a symmetric distribution, the multi-period return distribution could be asymmetric. The extent of asymmetry varies by asset class, with a plot of implied volatilities against the strike for equity index options showing a downward slope, or "volatility skew".

Engle (2011) develops a test for long-term skewness using a Merton (1974) style structural default model. Skewness in a market factor implies high defaults accompanied by substantial drops in equity prices and default correlations even far in the future corroborate the importance of long-term skewness. Thus, investors concerned about long-term risks should hedge exposure as in the ICAPM. An aversion towards negatively skewed returns implies many investors are willing to trade some of their average return for a decreased chance that they will experience a large reduction in their wealth. In keeping with this, Harvey, et al (2010) demonstrate the importance of considering higher-order moments that include skewness in the portfolio construction process.

Our own evaluation of common public market assets using monthly data from January 1992 to July 2021 is consistent with the aforementioned studies. Significant negative skewness is evident in selected equity markets, commodities, US High Yield bonds and REITs. In contrast, US Treasuries exhibit some degree of positive skewness and can be viewed as hedging assets with respect to negative skewness. Sourcing details for the included asset classes can be found in the Appendix.

| Table 1: Historical Summary of Asset Class Return Outcomes: Jan 1992 – July 2021 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Intermediate US Treasuries | Long US Treasuries | US Aggregate Bonds | US High Yield Bonds | US Large Cap Equities | EAFE Equities | Emerging Market Equities | US REITs | Commodities |
| Annualized Return | 4.4% | 7.2% | 5.3% | 7.6% | 10.0% | 5.6% | 7.6% | 10.1% | 2.7% |
| Standard Deviaton | 3.0% | 10.2% | 3.5% | 8.3% | 14.4% | 16.1% | 21.9% | 19.4% | 14.7% |
| Skew | 0.10 | 0.24 | -0.19 | -1.31 | -0.65 | -0.53 | -0.65 | -0.77 | -0.51 |
| Skew Significance | 0.78 | 1.86 | -1.45 | -8.09 | -4.68 | -3.90 | -4.68 | -5.43 | -3.81 |
| Kurtosis | 0.63 | 1.27 | 0.74 | 10.53 | 1.42 | 1.40 | 2.10 | 8.28 | 2.20 |
| Kurtosis Significance | 2.14 | 3.42 | 2.39 | 8.75 | 3.66 | 3.63 | 4.58 | 8.15 | 4.70 |
| Maximum Drawdown | 3.4% | 19.4% | 5.1% | 33.3% | 50.9% | 56.7% | 61.4% | 70.5% | 72.0% |
| Sharpe Ratio | 0.69 | 0.52 | 0.84 | 0.67 | 0.60 | 0.28 | 0.35 | 0.50 | 0.09 |
| Sortino Ratio | 2.72 | 1.23 | 2.67 | 1.34 | 1.12 | 0.59 | 0.64 | 0.87 | 0.32 |
| DD on Stdev | 1.16 | 1.90 | 1.46 | 4.02 | 3.53 | 3.52 | 2.81 | 3.63 | 4.91 |

*Source: PGIM Quantitative Solution. Net total return as of July 31, 2021.*

| Table 2: Representative Benchmark Portfolio Weights and Historical Return Characteristics | | | |
|---|---|---|---|
| Assets | Benchmark Weight | Benchmark Characteristics | |
| Intermediate US Treasuries | 5% | Annualized Return | 7.8% |
| Long US Treasuries | 5% | Standard Deviaton | 9.2% |
| US Aggregate Bonds | 20% | Skew | -0.95 |
| US High Yield Bonds | 5% | Skew Significance | -6.42 |
| US Large Cap Equities | 35% | Kurtosis | 3.48 |
| EAFE Equities | 15% | Kurtosis Significance | 5.88 |
| Emerging Market Equities | 5% | Shortfall | 29.6% |
| US REITs | 5% | Maximum Drawdown | 37.7% |
| Commodities | 5% | Sharpe Ratio | 0.64 |
| | | Sortino Ratio | 1.28 |
| | | DD on Stdev | 4.08 |
| Asset Groups | | | |
| Equity | 55% | | |
| Fixed Income | 35% | | |
| Alternatives | 10% | | |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Table 1 gives a summary of the historical monthly performance of various asset classes for the period January 1992-July 2021. Table 2 depicts a benchmark portfolio of hypothetical public holdings of a representative asset owner. Consistent with asset pricing theory based on the CAPM, assets with higher volatility generally have realized stronger absolute returns, although commodities are a notable exception. Looking beyond the traditional return and volatility measures, we see that the higher-order risk measures in skewness and kurtosis are significant across several of the component asset classes. If financial market return distributions followed a standard normal distribution consistent with average volatility outcomes, occurrences of "100-year events" should indeed be rare. However, history does not bear this out. Since 1921, US Large Cap stocks have experienced three such 100-year events (on an annual basis), which corresponded to more than three-sigma downside variation implied from the full sample volatility measure. On a monthly basis, between 1992-2021, there have been eight three-sigma events – double the roughly four expected events if returns were normally distributed and consistent with the full sample volatility measure. Missing in these observations is the well-established evidence that asset volatilities are not static but time varying. However, for the standard mean-variance optimization problem it is most common to use long-term average expected risk parameters that will not account for episodic regimes of elevated volatility.

Equity returns – US Large Cap, EAFE and Emerging Markets – display significant negative skewness and excess kurtosis. Real assets such as REITs and commodities, as well as hybrid asset classes such as US High Yield bonds, also display similar negatively skewed and fatter tail profiles. Fixed income asset classes such as US Aggregate bonds as well as Intermediate and Long-Term US Treasuries experienced a long-term bull market during the sample period as yields broadly declined with neutral- to modestly-positive skewness. Consequently, we see these "safe" asset classes having the lowest risk-adjusted drawdowns and the "risky" asset classes showing significant risk-adjusted drawdowns in this historical period as captured by the maximum drawdown adjusted for standard deviation or DD on stdev presented in the last row of Table 1.

Given its constituent parts, the outcomes for the representative benchmark shown in Table 2 are not surprising, although an elevated skew parameter larger than all but one of the included asset classes is indicative of positive co-skewness in the multi-asset portfolio.

Private assets, which are not covered here given the unavailability of comparable mark to market data, present asset owners with additional risk management considerations as the uncertain nature of cash flows from capital calls may result in the need to modify allocations to liquid assets to accommodate this intertemporal risk. For a more detailed evaluation of these considerations please see Shen et al. (2019).

## II. Approaches to Managing Multi-Dimensional Risk in Strategic Portfolio Construction

In this section, we examine a range of options for building strategic multi-asset portfolio allocations considering the multi-dimensional risk considerations outlined thus far. Portfolio construction will incorporate managing portfolio risk in three dimensions: 1) What are the outcomes under MVO and alternative risk models that consider equal risk contribution, historical drawdowns, historical shortfall to target and portfolio higher moments; 2) How ex-ante constraints on asset classes and asset class groups impact those outcomes; and finally, 3) How increasing the opportunity set with a risk hedging defensive equity allocation impacts outcomes.

### Review of Risk-Based Algorithms for Constructing Portfolios

In this section we present a taxonomy of risk metrics used in alternative portfolio construction methods. The metrics considered here fall into two broad categories: those that measure individual period risk (such as portfolio variance) and those that measure cumulative risk (such as maximum drawdown). Since portfolio return processes with high variance tend to have higher drawdowns and vice versa, choosing the appropriate metric is difficult and requires a deeper analysis of the asset return process. For example, if an asset exhibits positive serial correlation (i.e. if the asset appreciated last month, then it is more likely to appreciate next month), then the likely drawdowns would be larger than if the process had zero or negative serial correlation. In this case, the investor would prefer a risk metric that focuses more on drawdown than on variance. On the other hand, if the processes exhibit negative serial correlation, then drawdown is less of a concern and the investor is better off with improved diversification.

Standard deviation, or variance of the portfolio returns, is arguably the most common portfolio risk metric and is the foundation of the mean-variance optimization (MVO) formulation introduced by Markowitz (1952). Implicit in this formulation is the assumption of period-to-period independence and elliptically symmetric (generalized Gaussian) returns. MVO portfolios are generally well diversified since they account both for the asset volatilities and the cross-asset correlations. MVO places no weight on asymmetric return distribution (e.g. options skew) or serial correlations. As such, it has material shortcomings as a guide for choosing among options portfolios or actively traded portfolios. However, due to its general acceptance, it serves as the benchmark for many of the results highlighted in this paper.

MVSK (mean-variance-skew-kurtosis) is an extension of MVO that includes the higher-order moments of skew and kurtosis into the optimization process. Skew measures the asymmetry of the return distribution, as found, for example, in call option returns. Long call options exhibit positive skew where the downside is limited to the premium paid and the upside can be unlimited, and conversely for short call options that exhibit negative skew. In terms of risk-controlled portfolios, positive skew is preferred to negative skew. Kurtosis measures the relative likelihood of extreme (positive or negative) returns compared to a standard normal. Like large variances, these “fat tails” are generally to be avoided. Until recently the non-convexity of both the skew and kurtosis surfaces prevented their inclusion in standard optimization packages; however, recent work by Zhou and Palomar (2021) in finding tight convex approximations led to the MVSK results highlighted here.

Risk-parity, or “equal risk”, portfolios are constructed by weighting each asset with an inverse proportion to the standard deviation of its return. Expected asset return plays no role here; this is because all asset choices are assumed to have reasonably similar risk-adjusted returns regardless of the asset or its correlation with other assets. This arguably imparts a robustness advantage as errors associated with noisy mean estimation are reduced. Compared with an MVO portfolio, a risk-parity portfolio will have a greater weight on low-volatility assets, and a weaker reliance on diversification. It is interesting to note that this strategy performed particularly well in the 2007-2009 market meltdown and during the extended bond bull market beginning in the mid-1980s. A hybrid of MVO and risk-parity is the minimum variance portfolio that incorporates the full correlation structure of the assets, but ignores the assets’ expected returns in the optimization. As the results for minimum variance are expected to fall between risk-parity and MVO, it is excluded for clarity of presentation. Further discussion of the comparability of risk-parity portfolios to those constructed using MVO can be found in Kaya and Lee (2012).

Value-at-Risk (VaR), Conditional VaR (CVaR) and Mean Shortfall Optimization (MSO) all focus on single period negative outcomes. The investor specifies a percentile rank that divides the “good” returns from the “bad”. VaR is defined as the return outcome corresponding to that percentile rank. (e.g. if 95% of the returns are above -10%, then the 95th percentile VaR is -10%). Though VaR reports the worst losses that can be expected with a certain probability, it does not report on the extent of those losses. If 95% of outcomes are more favorable than -10%, VaR offers no insight how bad those other 5% of outcomes can be. As a result of this non-specificity, VaR is a bad choice for an optimization risk-metric. Since many portfolio allocation choices would result in the same VaR, the optimization cannot reliably find a downhill direction to search for a “better” allocation. A derivation of the MSO optimization method and comparability to other VaR based methods is presented in Bertimas et. al (2004).

MSO and CVaR work with the same set of negative outcomes as VaR, but both average the returns in that set to give a measure of the magnitude of negative outcomes. The averaging process incorporates a range of shortfall outcomes, harnessing a greater breadth of information than VaR, which only incorporates the single-most adverse event in the data sample. The difference between the two is that MSO averages the distances from the expected portfolio mean to the negative outcomes, whereas CVaR typically averages from zero to the negative outcome returns. In the cases presented here, we often target the expected return (e.g. build a portfolio with an 8% return target) and minimize the risk. In this case, CVaR and MSO yield identical results, so only MSO is presented in the next section. Generally speaking the CVaR/MSO portfolios look fairly similar to MVO portfolios, with deviations occurring when there are significantly asymmetric asset distributions.

Maximum Drawdown (maxDD), as the name implies, captures the worst cumulative loss an investor would endure over the historical period studied (in-sample). Finding a portfolio to minimize this metric often involves finding the lowest-risk assets (e.g. US Treasury bills) which can lead to low expected returns and poor diversification. Furthermore, since a maximum drawdown is a singular event, optimizing for maxDD in-sample often leads to poor out-of-sample behavior, as the singular event in-sample is unlikely to reoccur in exactly the same manner out-of-sample.

A variant that combines the benefits of CVaR/MSO and maxDD is Conditional Drawdown-at-Risk (CDaR) as developed in Chekhlov (2003). In the same spirit as CVaR, it averages the worst drawdown outcomes as a risk metric. Since it considers several drawdown outcomes, it is more robust out of sample than maxDD, and like CVaR it allows an efficient solution for the best allocation. Absent other business constraints, CDaR portfolios tend to have more concentrated asset holdings than MVO.

## III. Review of In-Sample Portfolio Results

In this section we present in-sample historical results for four portfolio construction optimization methods targeting different metrics of risk and evaluate the outcomes under different portfolio constraints. For each portfolio constructed we will target a return consistent with the historical representative benchmark return over the sample period of January 1992 through July 2021. Depending on the portfolio construction method, risk parameters will be drawn from the same historical sample of monthly returns. As such, these are perfect foresight portfolios and not representative of more realistic portfolios built with forward-looking assumptions that will be covered in the next section.

The risk-based portfolio construction methods to be evaluated are:

Mean Variance Optimization (MVO), Mean Shortfall Optimization (MSO), Conditional Drawdown at Risk (CDaR) and Mean Variance Skewness Kurtosis Optimization (MVSK). We also evaluate the properties of an equal risk-based portfolio with a leverage limit set to approximate a realized risk comparable to the other candidate portfolio construction methods.

We evaluate each of the portfolio construction methods under three sets of constraints: unbounded, asset class group constrained and asset class group and individual asset class constrained. By doing so, we will be able to evaluate the extent to which imposing ex-ante constraints limits the efficacy of each method with perfect foresight, acknowledging that diversification constraints are a common means of managing risk in the presence of uncertain future outcomes. For the unbounded problem, no asset or asset group constraints are applied. For the group constrained problem, we impose minimum and maximum asset class group constraints ranging from 30% to 70% for both fixed income and equities and a range of 5% to 20% for real assets. For the group and asset class constrained problem, we tighten the group constraints further and impose minimum position constraints for the largest benchmark allocations to US and International Equities as well as US Aggregate Bonds.

Allocations for the unbounded constructed portfolios are in Table 3a. Included for reference are the benchmark and equal risk allocations. For each of the risk-based methods, all allocations show a significant deviation from benchmark allocations towards more fixed income assets.

**Table 3a: In Sample - Weights Across Strategies - Unbounded**

| Assets | Benchmark | MVO | MVSK | MSO | CDaR | Eq Risk |
|---|---|---|---|---|---|---|
| Intermediate US Treasuries | 5% | 9% | 11% | 8% | 14% | 44% |
| Long US Treasuries | 5% | 38% | 48% | 45% | 54% | 13% |
| US Aggregate Bonds | 20% | 0% | 0% | 0% | 0% | 37% |
| US High Yield Bonds | 5% | 29% | 14% | 23% | 1% | 16% |
| US Large Cap Equities | 35% | 23% | 27% | 23% | 31% | 9% |
| EAFE Equities | 15% | 0% | 0% | 0% | 0% | 8% |
| Emerging Market Equities | 5% | 0% | 0% | 0% | 0% | 6% |
| US REITs | 5% | 1% | 0% | 0% | 0% | 7% |
| Commodities | 5% | 0% | 0% | 0% | 0% | 9% |
| **Asset Groups** | | | | | | |
| Equity | 55% | 23% | 27% | 23% | 31% | 23% |
| Fixed Income | 35% | 76% | 73% | 77% | 69% | 111% |
| Alternatives | 10% | 1% | 0% | 0% | 0% | 16% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

In-sample historical outcomes for the constructed unbounded portfolios are presented in table 3b. With the benefit of perfect foresight, the constructed portfolios all improve on the benchmark outcomes with respect to reduced volatility and drawdowns. The MVSK and CDaR risk models, which penalize negative skewness and drawdowns respectively, show the most improvement in reducing the higher moments of the returns distribution as well as limiting drawdowns. The MSO portfolio outcomes are comparable to those of MVO with a somewhat larger drawdown. The Equal Risk portfolio, which only considers the volatility of each asset class, had the strongest historical performance of the sample period as measured by Sharpe ratio, though experienced the largest drawdown and exhibited the largest deviation from normality of the non-benchmark portfolios. It is worth noting that over this historical period, Equal Risk portfolios with a substantial weighting in fixed income assets benefited greatly from the secular decline in interest rates. An outcome unlikely to be repeated looking forward given the current historically low interest rate environment.

Thus, even with a perfect knowledge of historical asset class performance characteristics, investors constructing portfolios emphasizing different risk considerations can end up choosing very different portfolios that reflect their preferences.

**Table 3b: In Sample - Performance Across Risk Models - Unbounded**

| Assets | Benchmark | MVO | MVSK | MSO | CDaR | Eq Risk |
|---|---|---|---|---|---|---|
| Annualized Return | 7.8% | 8.1% | 8.1% | 8.1% | 8.0% | 9.3% |
| Standard Deviaton | 9.2% | 6.1% | 6.3% | 6.2% | 6.6% | 7.0% |
| Skew | -0.95 | -0.73 | -0.47 | -0.57 | -0.36 | -1.07 |
| Skew Significance | -6.42 | -5.21 | -3.54 | -4.21 | -2.73 | -6.99 |
| Kurtosis | 3.48 | 3.55 | 1.78 | 2.47 | 1.35 | 5.45 |
| Kurtosis Significance | 5.88 | 5.94 | 4.18 | 4.99 | 3.54 | 7.07 |
| Shortfall | 29.6% | 20.5% | 20.5% | 20.3% | 21.2% | 24.3% |
| Maximum Drawdown | 37.7% | 16.1% | 13.1% | 13.9% | 11.6% | 24.1% |
| Sharpe Ratio | 0.64 | 0.97 | 0.94 | 0.96 | 0.89 | 1.03 |
| Sortino Ratio | 1.28 | 1.54 | 1.53 | 1.55 | 1.46 | 1.61 |
| DD on Stdev | 4.08 | 2.63 | 2.09 | 2.25 | 1.74 | 3.44 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Portfolio allocations for the four risk models under asset class group constraints are presented in Table 4a. Notable is that even with fairly generous constraints, the risk models each allocate very similarly.

**Table 4a: In Sample - Weights Across Strategies - Group Constrained**

| Assets | MVO | MVSK | MSO | CDaR |
|---|---|---|---|---|
| Intermediate US Treasuries | 21% | 20% | 20% | 7% |
| Long US Treasuries | 33% | 40% | 41% | 58% |
| US Aggregate Bonds | 0% | 0% | 0% | 0% |
| US High Yield Bonds | 11% | 4% | 4% | 0% |
| US Large Cap Equities | 30% | 30% | 30% | 30% |
| EAFE Equities | 0% | 0% | 0% | 0% |
| Emerging Market Equities | 0% | 0% | 0% | 0% |
| US REITs | 5% | 5% | 5% | 0% |
| Commodities | 0% | 0% | 0% | 5% |
| **Asset Groups** | | | | |
| Fixed Income | 65% | 65% | 65% | 65% |
| Equity | 30% | 30% | 30% | 30% |
| Real Assets | 5% | 5% | 5% | 5% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Portfolio outcomes for the group constrained portfolios are presented in Table 4b. Not surprisingly, given the similar allocations, there is less differentiation in outcomes among the different risk models. Only CDaR, with a somewhat higher allocation to Long Treasuries, has materially higher skew and an improved drawdown.

**Table 4b: In Sample - Performance Across Risk Models - Group Constraints**

| Assets | MVO | MVSK | MSO | CDaR |
|---|---|---|---|---|
| Annualized Return | 8.1% | 8.1% | 8.1% | 8.0% |
| Standard Deviaton | 6.2% | 6.3% | 6.3% | 6.8% |
| Skew | -0.69 | -0.60 | -0.59 | -0.45 |
| Skew Significance | -4.97 | -4.37 | -4.32 | -3.38 |
| Kurtosis | 2.90 | 2.34 | 2.30 | 1.77 |
| Kurtosis Significance | 5.40 | 4.85 | 4.81 | 4.16 |
| Shortfall | 20.8% | 20.6% | 20.6% | 21.8% |
| Maximum Drawdown | 18.6% | 16.6% | 16.4% | 13.5% |
| Sharpe Ratio | 0.95 | 0.94 | 0.93 | 0.87 |
| Sortino Ratio | 1.51 | 1.51 | 1.50 | 1.40 |
| DD on Stdev | 2.98 | 2.63 | 2.60 | 1.98 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Allocations and outcomes for portfolios constructed with both group and minimum individual asset class position constraints are presented in Tables 5a and 5b. Here, the impact of constraints to differentiate the methods is even more pronounced, resulting in virtually identical allocations and portfolio outcomes.

From these summary results, we find that for those looking to explore the potential benefits of portfolio construction methods (other than MVO), there is a discovery process that should begin with a relatively unconstrained proposition as the imposition of tighter constraints around benchmark allocations will not yield differentiated portfolios.

**Table 5a: In Sample - Weights Across Strategies - Group and Position Constrained**

| Assets | MVO | MVSK | MSO | CDaR |
|---|---|---|---|---|
| Intermediate US Treasuries | 9% | 9% | 9% | 9% |
| Long US Treasuries | 10% | 10% | 10% | 10% |
| US Aggregate Bonds | 30% | 30% | 30% | 30% |
| US High Yield Bonds | 1% | 0% | 1% | 1% |
| US Large Cap Equities | 35% | 41% | 35% | 35% |
| EAFE Equities | 5% | 5% | 5% | 5% |
| Emerging Market Equities | 0% | 0% | 0% | 0% |
| US REITs | 10% | 5% | 10% | 10% |
| Commodities | 0% | 0% | 0% | 0% |
| **Asset Groups** | | | | |
| Fixed Income | 50% | 49% | 50% | 50% |
| Equity | 40% | 46% | 40% | 40% |
| Real Assets | 10% | 5% | 10% | 10% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

**Table 5b: In Sample - Performance Across Risk Models - Group and Position Constrained**

| Assets | MVO | MVSK | MSO | CDaR |
|---|---|---|---|---|
| Annualized Return | 8.0% | 8.0% | 8.0% | 8.0% |
| Standard Deviaton | 7.3% | 7.4% | 7.3% | 7.3% |
| Skew | -0.86 | -0.74 | -0.86 | -0.86 |
| Skew Significance | -5.92 | -5.26 | -5.92 | -5.92 |
| Kurtosis | 3.49 | 2.50 | 3.50 | 3.50 |
| Kurtosis Significance | 5.89 | 5.02 | 5.89 | 5.89 |
| Shortfall | 23.9% | 24.0% | 23.9% | 23.9% |
| Maximum Drawdown | 28.6% | 28.0% | 28.6% | 28.6% |
| Sharpe Ratio | 0.80 | 0.80 | 0.81 | 0.81 |
| Sortino Ratio | 1.22 | 1.22 | 1.23 | 1.23 |
| DD on Stdev | 3.93 | 3.80 | 3.93 | 3.93 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

## Introducing Defensive Equity to the Opportunity Set

As mentioned earlier, another means to potentially manage higher dimensional risk in strategic portfolios is to introduce additional assets to hedge against those risks. Given the higher volatility and skewness present in equities, we evaluate the impact on outcomes of including a defensive equity allocation using PGIM Quantitative Solutions' US Market Participation Strategy.

PGIM Quantitative Solutions' US Market Participation Strategy (MPS) seeks to provide upside participation when the US equity market advances, while reducing downside risk. The strategy utilizes long-dated S&P 500 call options in combination with US Treasury bonds. Call options seek to provide upside participation, while US Treasury bonds serve as a safe haven during turbulent market conditions and provide downside protection. Using a disciplined process, exposures (market, volatility and duration) are actively managed in response to the changing market environment using a rules-based framework.

As presented in Table 6, in contrast to US Large Cap Equities, MPS provides a historical annualized return that is approximately 85% of the return of US Large Cap Equities with considerably lower volatility and modest positive skewness.

Table 7a and 7b present the allocations and in-sample historical outcomes for portfolios constructed with MPS under the group constraint set, with MPS added to the opportunity set as an off-benchmark equity option.

**Table 6: MPS vs. US Large Cap**

| | US Large Cap Equities | MPS |
|---|---|---|
| Annualized Return | 10.0% | 8.7% |
| Standard Deviaton | 14.4% | 8.7% |
| Skew | -0.65 | 0.18 |
| Skew Significance | -4.68 | 1.38 |
| Kurtosis | 1.42 | 3.51 |
| Kurtosis Significance | 3.66 | 5.91 |
| Shortfall | 43.8% | 25.0% |
| Maximum Drawdown | 50.9% | 19.0% |
| Sharpe Ratio | 0.60 | 0.77 |
| Sortino Ratio | 1.12 | 1.78 |
| DD on Stdev | 3.53 | 2.18 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

**Table 7a: In Sample - Weights Across Strategies - Group Constrained**

| Assets | MVO | MVSK | MSO | CDaR | Eq Risk |
|---|---|---|---|---|---|
| Intermediate US Treasuries | 6% | 0% | 10% | 14% | 40% |
| Long US Treasuries | 27% | 22% | 21% | 25% | 12% |
| US Aggregate Bonds | 0% | 15% | 0% | 0% | 34% |
| US High Yield Bonds | 27% | 0% | 18% | 1% | 15% |
| US Large Cap Equities | 0% | 0% | 0% | 0% | 8% |
| EAFE Equities | 0% | 0% | 0% | 0% | 7% |
| Emerging Market Equities | 0% | 0% | 0% | 0% | 5% |
| US REITs | 5% | 4% | 5% | 6% | 6% |
| Commodities | 0% | 1% | 0% | 0% | 8% |
| MPS | 36% | 58% | 46% | 54% | 14% |
| **Asset Groups** | | | | | |
| Fixed Income | 59% | 37% | 49% | 40% | 101% |
| Equity | 36% | 58% | 46% | 54% | 35% |
| Real Asset | 5% | 5% | 5% | 6% | 14% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

| Table 7b: In Sample - Performance Across Risk Models - Group Constraints | | | | | |
|---|---|---|---|---|---|
| Assets | MVO | MVSK | MSO | CDaR | EQ Risk |
| Annualized Return | 8.1% | 8.1% | 8.1% | 8.1% | 9.7% |
| Standard Deviaton | 5.6% | 6.1% | 5.7% | 6.0% | 7.1% |
| Skew | -0.25 | 0.23 | -0.06 | 0.17 | -0.84 |
| Skew Significance | -1.93 | 1.78 | -0.46 | 1.33 | -5.79 |
| Kurtosis | 1.31 | 2.43 | 1.49 | 2.16 | 3.39 |
| Kurtosis Significance | 3.48 | 4.95 | 3.77 | 4.65 | 5.81 |
| Shortfall | 18.6% | 19.1% | 18.4% | 19.0% | 24.5% |
| Maximum Drawdown | 10.6% | 7.5% | 9.7% | 7.4% | 22.6% |
| Sharpe Ratio | 1.06 | 0.97 | 1.04 | 0.98 | 1.07 |
| Sortino Ratio | 1.82 | 1.72 | 1.82 | 1.74 | 1.72 |
| DD on Stdev | 1.91 | 1.23 | 1.71 | 1.23 | 3.20 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Adding MPS to the asset universe materially improves portfolio outcomes for all evaluated risk models. Most notably, each of the algorithms allocate all equity exposure to MPS, resulting in portfolios with lower volatility, reduced drawdowns and a substantial increase in skewness parameters to positive and modestly significant levels for the MVSK and CDaR risk models. Sharpe ratios of all risk model portfolios are improved to near or above 1 with no reduction in annualized return. The Equal Risk portfolio continues to show strong annualized performance relative to the risk model optimized portfolios, although risk-adjusted drawdowns and negative skewness are considerably higher.

Thus, investors with access to a tail-risk hedged product such as MPS are able to construct portfolios that performed competitively with Equal Risk portfolios historically, while also improving the risk measures which they focus on, such as Sharpe Ratio, drawdowns and return symmetry.

## IV. Out of Sample Portfolio Evaluation

The more realistic exercise in building strategic portfolios is to incorporate forward-looking expectations based on initial conditions. The importance of incorporating initial vs. historical conditions in the current environment is best exemplified by fixed income assets which benefited from a long secular decline in underlying government interest rates over the historical period detailed in Section III, but would be unexpected to perform as well with the low interest rate environment prevailing today. Other considerations, such as relative valuation levels and reasonable expectations for growth and inflation, can also importantly deviate from the historical experience.

PGIM Quantitative Solutions' Capital Market Assumptions (CMAs) underpin the long-run outlook for strategic allocations in our individual strategies and multi-asset portfolios. They are the product of a highly systematic process for generating consistent projections across the capital markets.

CMAs provide 10-year expectations for the most widely held equity, fixed income and non-traditional asset classes, measuring both return and risk. We update our CMAs each quarter. Our investment professionals begin with evolving asset class fundamentals and macroeconomic assumptions at the country level. For each asset class we decompose local return expectations into three broad categories: income, growth and valuation adjustment. We also forecast relative currency adjustments for investors in different domiciles to allow for conversion to hedged or unhedged returns. Our core building blocks and final forecasts are reviewed at their component levels by an investment council of our most senior investment professionals.

Our latest CMAs for the subset of asset classes in this analysis are presented in Table 8. Compared to the earlier historical period reviewed, our current CMAs are forecasting more modest outcomes for the next ten years, attributable to the current lower yield environment, more modest expectations for economic growth and inflation as well as elevated valuations for US equities relative to long-term averages.

**Table 8: Summary of Q3 2021 CMAs**

| Assets | Arithmetic Mean | Geometric Mean | Standard Deviation | Sharpe Ratio |
|---|---|---|---|---|
| Intermediate US Treasuries | 1.3% | 1.2% | 3.0% | 0.30 |
| Long US Treasuries | 3.2% | 2.7% | 10.2% | 0.28 |
| US Aggregate Bonds | 2.3% | 2.2% | 5.6% | 0.35 |
| US High Yield Bonds | 3.3% | 2.9% | 8.5% | 0.34 |
| US Large Cap Equities | 6.4% | 5.2% | 15.1% | 0.40 |
| EAFE Equities | 8.0% | 6.7% | 16.0% | 0.48 |
| Emerging Market Equities | 9.4% | 6.7% | 23.6% | 0.38 |
| US REITs | 6.3% | 4.8% | 17.4% | 0.34 |
| Commodities | 2.4% | 1.3% | 14.6% | 0.14 |
| MPS | 4.8% | 4.4% | 8.7% | 0.51 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Portfolios constructed for the various portfolio construction methods using group constraints and current CMAs are presented in Table 9. For each optimization method, the expected return on the benchmark portfolio is targeted. Risk parameters for MVSK, MSO and CDaR are estimated from the historical sample. In comparison to the perfect foresight portfolios constructed in Section III, portfolios constructed based on the current CMAs find all risk models allocating materially to both MPS and Long Treasuries. Other allocations across fixed income and equities vary by risk model.

**Table 9: Optimized allocations using current CMAs**

| Assets | Benchmark | MVO | MVSK | MSO | CDaR |
|---|---|---|---|---|---|
| Intermediate US Treasuries | 5% | 0% | 0% | 0% | 0% |
| Long US Treasuries | 5% | 32% | 30% | 30% | 44% |
| US Aggregate Bonds | 20% | 6% | 0% | 0% | 0% |
| US High Yield Bonds | 5% | 0% | 0% | 0% | 0% |
| US Large Cap Equities | 35% | 0% | 44% | 0% | 0% |
| EAFE Equities | 15% | 19% | 0% | 16% | 0% |
| Emerging Market Equities | 5% | 8% | 2% | 6% | 21% |
| US REITs | 5% | 5% | 5% | 5% | 5% |
| Commodities | 5% | 0% | 0% | 0% | 0% |
| MPS | 0% | 31% | 19% | 43% | 30% |
| **Asset Groups** | | | | | |
| Fixed Income | 35% | 37% | 30% | 30% | 44% |
| Equities | 55% | 58% | 65% | 65% | 51% |
| Real Assets | 10% | 5% | 5% | 5% | 5% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

## Regime Aware Forward-Looking Simulations

A common exercise to evaluate potential strategic portfolio outcomes is to conduct forward-looking simulations informed by expected arithmetic returns and a covariance matrix. In most cases these simulations assume a multivariate normal distribution of asset class returns that is inconsistent with observed historical outcomes.

To build more robust simulations that consider periods of crisis that result in more pronounced drawdowns than would be captured in static average expected return and covariance forecasts, we develop a simulation methodology that switches between expansion and recession regimes via a Markov process. For each regime we sample monthly returns from January 1992 to July 2021 based on the National Bureau of Economic Research (NBER) business cycle dating for US economic expansions and recessions. Historical outcomes for expansionary and recessionary periods are presented in Table 10.

**Table 10: Historical Annualized Returns and Standard Deviations in Expansion and Recession Regimes**

| | Expansion | | Recession | |
|---|---|---|---|---|
| | Expected Return | Standard Deviation | Expected Return | Standard Deviation |
| Commodities | 4.89% | 13.05% | -26.41% | 26.18% |
| EAFE Equities | 10.70% | 14.46% | -23.50% | 27.91% |
| Emerging Market Equities | 11.76% | 19.89% | -17.65% | 37.89% |
| Intermediate US Treasuries | 1.04% | 2.81% | 3.85% | 4.53% |
| Long US Treasuries | 3.01% | 9.72% | 5.75% | 14.91% |
| MPS | 5.87% | 8.87% | -7.32% | 6.17% |
| US REITs | 8.96% | 15.08% | -25.11% | 45.83% |
| US Aggregate Bonds | 2.26% | 3.42% | 3.07% | 4.77% |
| US High Yield Bonds | 4.21% | 6.13% | -7.64% | 20.88% |
| US Large Cap Equities | 8.69% | 12.92% | -20.71% | 25.35% |

*Source: PGIM Quantitative Solutions as of July 31, 2021.* ***There is no guarantee this objective will be met.***

The Markov process, in which a simulation will transition between expansionary and recessionary regimes, is calibrated to the historical NBER business cycle dates covering the period from 1945 to 2021. For this sample, the unconditional probability of being in an expansion for any month is 86%. Conditional on being in an expansion for any given month, the probability of staying in expansion the following month is 98%, equivalent to an expected duration of expansions of about 86 months. The corresponding probability of transitioning to a recession conditional on being in a recession is 90%, equivalent to an expected duration of recessions of 10 months. The transition probabilities of staying in the same regime or transitioning to another regime are presented in Table 11.

**Table 11. Conditional and Unconditional Probabilities of Regime Transition**

| | From | | Unconditional |
|---|---|---|---|
| Assets | Expansion | Recession | Probability |
| Expansion | 0.984 | 0.097 | 0.862 |
| Recession | 0.016 | 0.903 | 0.138 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

For the forward-looking simulations, expected returns for each asset in each regime are calculated by taking the difference in full sample returns for expansion and recession relative to the full sample unconditional returns and adding that differential to our existing CMA forecasts. Based on the transition probabilities, the simulation will switch between two joint normal conditional regimes. With this process, we simulate 1000 return paths for the assets, which are then used to evaluate the performance of the different constructed portfolios.

Presented in Table 12a are simulated asset class and representative benchmark outcomes assuming a single multivariate normal state. As expected by design, the reported simulation returns are consistent with the CMA inputs and there is no material skew or kurtosis. Table 12b presents the same results under the joint normal regime switching simulations. Here, the simulations show considerably higher drawdowns, about 30% higher for the benchmark, and directionally consistent higher moments more consistent with historical data.

**Table 12a: Simulation Results for Single Multivariate Normal Regime**

| Assets | Intermediate US Treasuries | Long US Treasuries | US Aggregate Bonds | US High Yield Bonds | US Large Cap Equities | EAFE Equities | Emerging Market Equities | US REITs | Commodities | MPS | Benchmark |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Arithmetic Return | 1.2% | 3.2% | 2.3% | 3.3% | 6.2% | 7.8% | 9.3% | 6.1% | 2.4% | 4.6% | 5.1% |
| Annualized Return | 1.2% | 2.7% | 2.2% | 3.0% | 5.2% | 6.5% | 6.9% | 4.2% | 1.3% | 4.2% | 4.7% |
| Standard Deviaton | 3.0% | 10.2% | 3.5% | 8.3% | 14.3% | 16.1% | 21.9% | 19.3% | 14.7% | 8.7% | 9.3% |
| Skew | 0.04 | 0.02 | 0.02 | -0.05 | 0.02 | -0.02 | 0.02 | 0.01 | 0.06 | 0.00 | 0.03 |
| Skew Significance | 0.17 | 0.10 | 0.10 | -0.23 | 0.09 | -0.09 | 0.09 | 0.04 | 0.26 | -0.02 | 0.13 |
| Kurtosis | -0.05 | -0.01 | 0.03 | 0.04 | -0.02 | -0.14 | -0.09 | -0.08 | 0.01 | -0.12 | -0.09 |
| Kurtosis Significance | -0.06 | 0.04 | 0.18 | 0.21 | 0.07 | -0.23 | -0.09 | -0.06 | 0.20 | -0.15 | -0.06 |
| Shortfall | 6.8% | 22.8% | 8.8% | 19.7% | 33.4% | 38.0% | 50.4% | 43.0% | 31.2% | 20.9% | 22.2% |
| Maximum Drawdown | 6.7% | 22.1% | 6.4% | 18.2% | 29.0% | 30.1% | 40.4% | 39.3% | 34.0% | 17.0% | 17.8% |
| Sharpe Ratio | 0.28 | 0.28 | 0.54 | 0.36 | 0.41 | 0.46 | 0.41 | 0.30 | 0.14 | 0.48 | 0.52 |
| Sortino Ratio | 0.43 | 0.43 | 0.87 | 0.54 | 0.65 | 0.74 | 0.64 | 0.46 | 0.20 | 0.77 | 0.83 |
| DD on Stdev | 2.26 | 2.17 | 1.82 | 2.17 | 2.03 | 1.88 | 1.84 | 2.04 | 2.31 | 1.95 | 1.92 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

**Table 12b: Simulation Results for Joint Normal Regime Aware Simulations**

| Assets | Intermediate US Treasuries | Long US Treasuries | US Aggregate Bonds | US High Yield Bonds | US Large Cap Equities | EAFE Equities | Emerging Market Equities | US REITs | Commodities | MPS | Benchmark |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Arithmetic Return | 1.33% | 3.41% | 2.31% | 2.94% | 5.38% | 6.95% | 8.78% | 5.54% | 1.31% | 4.09% | 4.65% |
| Annualized Return | 1.28% | 2.88% | 2.24% | 2.57% | 4.26% | 5.49% | 6.18% | 3.52% | 0.09% | 3.69% | 4.20% |
| Standard Deviaton | 3.04% | 10.33% | 3.54% | 8.32% | 14.86% | 16.93% | 22.58% | 19.60% | 15.54% | 8.86% | 9.38% |
| Skew | 0.12 | 0.03 | -0.03 | -0.35 | -0.31 | -0.24 | -0.16 | -0.45 | -0.27 | 0.09 | -0.33 |
| Skew Significance | 0.53 | 0.13 | -0.16 | -1.31 | -1.29 | -1.04 | -0.71 | -1.78 | -1.14 | 0.40 | -1.32 |
| Kurtosis | 0.19 | 0.26 | -0.10 | 3.87 | 1.11 | 1.03 | 0.60 | 3.28 | 1.06 | 0.05 | 2.02 |
| Kurtosis Significance | 0.48 | 0.65 | -0.10 | 3.13 | 1.57 | 1.42 | 0.98 | 2.94 | 1.71 | 0.19 | 2.26 |
| Shortfall | 7.0% | 23.8% | 8.9% | 21.8% | 37.4% | 43.4% | 55.1% | 51.6% | 36.3% | 20.6% | 25.1% |
| Maximum Drawdown | 6.4% | 21.9% | 6.4% | 19.7% | 36.2% | 37.2% | 45.0% | 42.3% | 42.6% | 18.9% | 23.2% |
| Sharpe Ratio | 0.32 | 0.30 | 0.55 | 0.33 | 0.34 | 0.39 | 0.38 | 0.28 | 0.06 | 0.42 | 0.47 |
| Sortino Ratio | 0.50 | 0.46 | 0.89 | 0.49 | 0.51 | 0.60 | 0.58 | 0.41 | 0.09 | 0.68 | 0.72 |
| DD on Stdev | 2.11 | 2.12 | 1.81 | 2.33 | 2.42 | 2.17 | 1.98 | 2.15 | 2.72 | 2.13 | 2.43 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

The regime simulation methodology also indirectly incorporates the dynamics of co-skewness, especially between negatively skewed assets. For example, US equities that have a negative skew have a moderate correlation with commodities during expansions, but this correlation becomes more pronounced during recessionary periods. Similarly, US equity correlation with a relatively un-skewed asset such as US Aggregate bonds also turns positive during recessions, while being slightly negative in expansions. The benefit of adding a truly diversifying asset is seen in the case of US Large Cap equities with respect to MPS. The correlation with MPS is high during expansions, as expected, but almost halves during recessionary periods. Other select conditional correlations can be found in Table 13.

**Table 13: Sample Correlations in Expansionary and Recessionary Regimes**

| Cross Asset Correlations | | |
|---|---|---|
| Asset Pair | Expansion | Recession |
| US Large Cap to Emerging Markets Equities | 0.65 | 0.87 |
| US Large Cap to EAFE Equities | 0.76 | 0.92 |
| EAFE Equities to Emerging Markets Equities | 0.69 | 0.95 |
| **Average** | **0.70** | **0.91** |
| Commodities to US Large Cap | 0.28 | 0.49 |
| Commodities to Emerging Markets Equities | 0.39 | 0.66 |
| **Average** | **0.34** | **0.57** |
| US Aggregate Bonds to US Large Cap | -0.01 | 0.27 |
| US Aggregate Bonds to Emerging Markets Equities | -0.06 | 0.29 |
| US Aggregate Bonds to EAFE Equities | -0.03 | 0.37 |
| **Average** | **-0.04** | **0.31** |
| Long US Treasuries to US Large Cap | -0.22 | -0.11 |
| Intermediate US Treasuries to US Large Cap | -0.16 | -0.35 |
| Long US Treasuries to Emerging Markets Equities | -0.23 | -0.11 |
| Intermediate US Treasuries to Emerging Markets Equities | -0.18 | -0.38 |
| **Average** | **-0.20** | **-0.24** |
| MPS to US Large Cap | 0.83 | 0.45 |
| MPS to Emerging Markets Equities | 0.48 | 0.24 |
| MPS to US Aggregate Bonds | 0.18 | 0.09 |
| MPS to Intermediate US Treasuries | 0.11 | -0.01 |
| MPS to Long US Treasuries | 0.00 | 0.09 |
| **Average** | **0.32** | **0.17** |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

Summary simulation results depicting the average of the 50 simulation outcomes around the median of 1000 runs are presented in Table 14. As in the case of the in-sample group constrained portfolios, the simulated portfolios are similar in their outcomes, though all improve relative to the benchmark in Sharpe ratio and more limited drawdowns. In contrast to the historical period, the Equal Risk portfolio no longer shows an expected return advantage and continues to show elevated negative skew and kurtosis relative to the optimized portfolios.

**Table 14. Regime Aware Simulated Outcomes for Risk Based Portfolio Construction Methods**

| Assets | MVO | MVSK | MSO | CDaR | EQ Risk |
|---|---|---|---|---|---|
| Annualized Return | 4.43% | 4.27% | 4.34% | 4.52% | 4.31% |
| Standard Deviaton | 7.22% | 8.55% | 7.45% | 7.76% | 7.47% |
| Skew | -0.22 | -0.23 | -0.21 | -0.11 | -0.28 |
| Skew Significance | -0.95 | -0.97 | -0.94 | -0.51 | -1.18 |
| Kurtosis | 0.84 | 0.54 | 0.45 | 0.41 | 1.27 |
| Kurtosis Significance | 1.22 | 0.82 | 0.84 | 0.82 | 1.93 |
| Shortfall | 19.1% | 22.0% | 19.5% | 20.1% | 20.5% |
| Maximum Drawdown | 15.1% | 18.6% | 15.5% | 15.4% | 16.9% |
| Sharpe Ratio | 0.60 | 0.50 | 0.57 | 0.58 | 0.58 |
| Sortino Ratio | 0.96 | 0.78 | 0.91 | 0,93 | 0.90 |
| DD on Stdev | 2.06 | 2.15 | 2.06 | 1.98 | 2.22 |

*Source: PGIM Quantitative Solutions as of July 31, 2021.*

## V. Conclusion

In this paper we confirm the observation shown in previous studies that asset class returns and multi-asset portfolios of public assets are significantly non-normally distributed, with negative skewness and considerable drawdown risk beyond what can be summarized with the conventional metrics of return and standard deviation. Our evaluation of alternative methods to construct strategic multi-asset portfolios, taking into consideration higher moments of asset class returns and measures of drawdown, show some efficacy in insulating portfolios against risks introduced by non-Gaussian return distributions, though this efficacy is limited in the presence of ex-ante asset class or asset class group constraints. Drawdown risk and negative skewness can also be mitigated through the introduction of additional hedging assets, such as a defensive equity allocation. Further, to better evaluate the potential future path of strategic portfolio outcomes in the presence of negative skewness, we introduce a regime-aware simulation methodology that more realistically reproduces the historical characteristics of asset classes and portfolio outcomes. These additional tools provide asset owners further breadth and insights into the important task of building long-horizon strategic portfolios to meet long-term funding requirements that incorporate risk beyond the conventional metrics of mean and variance.

## Appendix

| Assets | Index | Source |
|---|---|---|
| Intermediate US Treasuries | Bloomberg US Intermdiate Treasury TR Index | Bloomberg |
| Long US Treasuries | Bloomberg US Long Treasury TR Index | Bloomberg |
| US Aggregate Bonds | Bloomberg US Aggregate TR Index | Bloomberg |
| US High Yield Bonds | Bloomberg US Corporate High Yield Bond TR Index | Bloomberg |
| US Large Cap Equities | S&P 500 TR Index | Bloomberg |
| EAFE Equities | MSCI EAFE TR Index | Bloomberg |
| Emerging Market Equities | MSCI Emerging Markets TR Index | Bloomberg |
| US REITs | Dow Jones US Select REIT TR Index | Bloomberg |
| Commodities | Bloomberg Commodity TR Index | Bloomberg |

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