PGIM QUANTITATIVE SOLUTIONS

REGIME CONDITIONAL REVERSE STRESS TESTING

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Overview

- Regime conditional reverse stress tests help identify hidden scenarios that could have an adverse effect on a diversified multi-asset portfolio in different states of the economy.
- Reverse stress tests identify plausible asset class return outcomes given a specified adverse portfolio event and can complement traditional stress tests which reveal portfolio exposure to specific shocks.
- Conditional or stressed covariance matrices can be integrated into reverse stress test analysis. An example of how a reverse stress test can be used to extract scenarios is explored here by sampling data from historical periods of elevated inflation.

Stress tests have become an integral component of risk management (see [2] for a discussion of flexible risk platforms). Stress tests are more transparent and intuitive than risk estimates and are, therefore, essential complements to risk model analysis. Though risk models provide useful decompositions of portfolio risk, stress tests overcome the shortcomings of risk models by estimating the impact of adverse market movements on a portfolio, capturing volatility jumps and changing correlation structures, and incorporating liquidity shifts.

Reverse stress tests (RSTs) provide a complementary analysis to regular or traditional stress tests. Under a traditional stress test, risk factor shifts (scenarios) are specified as inputs and the loss of the portfolio is computed. Instead, reverse stress tests work in the opposite direction. Under a reverse stress test, we specify the portfolio loss as an input, and then find scenarios that can generate this loss (see Figure 1 for an illustration). For instance, what plausible combination of asset return outcomes would correspond with a total multi-asset portfolio loss of 10%?

It should be noted that regulators have endorsed RST analysis because of its ability to identify hidden risk factor scenarios that could threaten the survival of an institution (see [4] and [5]). While regulators and financial institutions placed more focus on stress testing after the Global Financial Crisis, reverse stress testing has gained widespread acceptance as an important risk management methodology. According to a report from the Basel Committee on Banking Supervision [4], reverse stress testing is conducted as a complimentary stress test by two-thirds of the institutions that they surveyed.

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In this note we explore an example of how RSTs can be used to extract scenarios from a high inflation environment. The inflation example builds from the work in the recent PGIM Quant paper Portfolio Implications of a Higher US Inflation Regime. In this paper, the authors acknowledge that there is a non-trivial probability that inflation could stay elevated for the next few years. In addition, they discuss the drivers and implications of higher inflation across different asset classes such as equities, bonds, commodities, and real estate and demonstrate that real assets can mitigate inflation risk. Here we utilize a conditional covariance matrix of asset returns extracted from high inflation periods.

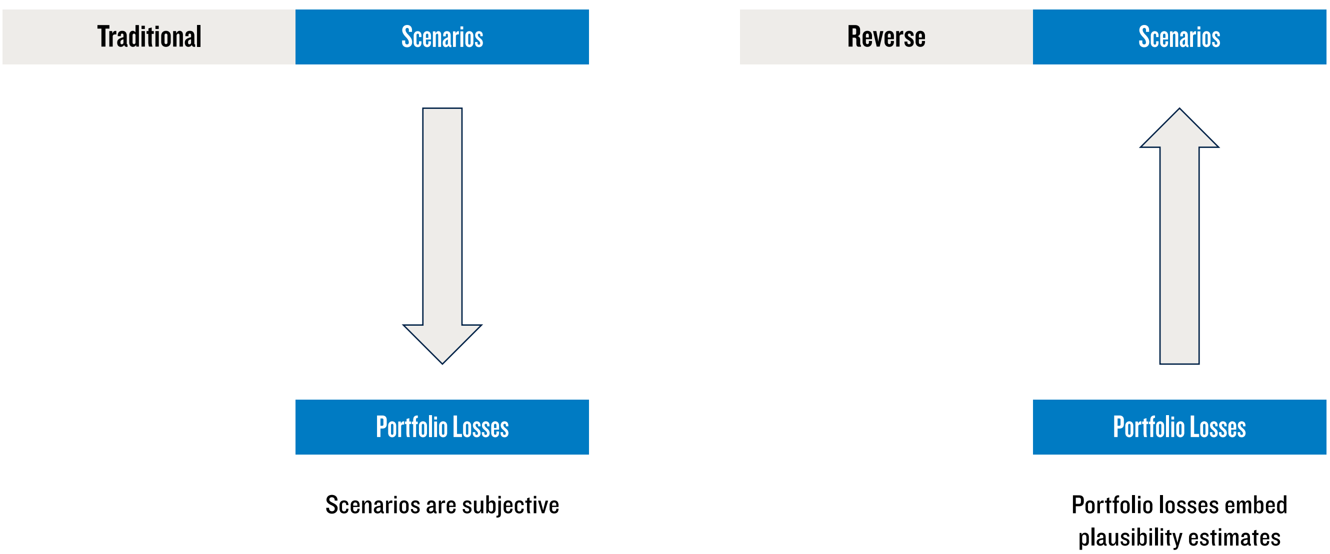
RST Requirements

Although the concept of reverse stress testing is simple and intuitive, the implementation of reverse stress testing can be computationally challenging. One such challenge is the generation of the scenarios, which should be severe but always plausible. Another challenge can be the computational cost of repricing securities for these scenarios, especially if positions exhibit nonlinear payoffs (See [3] or [4], for example).

Table 1 lists the modeling requirements for both traditional and reverse stress tests. Under an RST, the adverse impact on the portfolio is modeled by simply stating the portfolio loss as an input. Whereas the second criteria, plausibility, is subjective under a traditional stress test, a distributional model is required to estimate the likelihood of scenarios under an RST. Finally, we require that the scenarios are exhaustive, i.e., generate a sufficient number of scenarios to reveal hidden scenarios that severely impact the portfolio P&L. Typically, the exhaustivity condition requires grid pricing that uniformly spans the loss region.

Note that traditional stress tests are extremely useful, and RSTs should not replace but complement them. We will provide an example later in this note where an RST is combined with a traditional stress test.

Figure 1: Traditional and Reverse Stress Tests



Source: PGIM Quantitative Solutions

Table 1: Traditional vs. Reverse Stress Test Requirements

Requirements	Traditional Stress Test	Reverse Stress Test
Adverse	Select risk factors that will have an adverse impact on a given portfolio.	Portfolio loss is the input.
Plausible	Is the subjective scenario plausible?	Are the (output) scenarios plausible? A distributional assumption for risk factors is required to answer this question.
Exhaustive		Ensure that a sufficient number of scenarios is extracted to reveal hidden scenarios. Uniform grid pricing is required in the space of risk factors.

Source: PGIM Quantitative Solutions

Modeling Assumptions/Mathematical Formulation

An RST amounts to picking a portfolio loss and identifying the scenarios that produce this loss. Once the scenarios are identified, the probabilities corresponding with each scenario are used to select the plausible ones. The probabilities are derived from the dependence structure assumed (or identified) for the risk factors, such as a multivariate normal distribution. Thus, as already highlighted, the distributional assumption is a key input as it determines both plausibility and scenario generation.

We will only discuss the parametric case and follow the treatment from [6]. (For a discussion of the nonlinear case, see [3] and [4]). Consider asset returns $r = (r_1, \ldots, r_n)^T$ which are multivariate normal with the covariance matrix C. The portfolio return is $r_p = w^T r$ where $w = (w_1, \ldots, w_n)^T$ are the weights of the assets.

Under a reverse stress test, we specify a loss L, and determine the scenarios r^* that lead to this loss:

$$L = -w^T r^*$$

The scenarios from this equation are represented as a hyperplane in R^n as $H = \{r : w^T r = -L\}$. Figure 2 illustrates the case for two assets and also highlights scenarios which are more plausible. We can also represent the portfolio loss as VaR (value-at-risk) at some confidence level α:

$$L = \mathrm{VaR}_\alpha = q_\alpha \sqrt{w^T C w} = q_\alpha \sigma_p$$

where q_α is the quantile of the standard normal distribution at level α and σ_p is the portfolio volatility.

Selecting the exhaustive and plausible scenarios r^*

The plausibility of scenarios will then be modeled using the probability density from the multivariate normal distribution of asset returns. The conditional distribution of r given that $r \in H$ is normally distributed with mean $\bar{r}_H$. It can be shown that the mean of this conditional distribution is equal to the gradient of VaR:

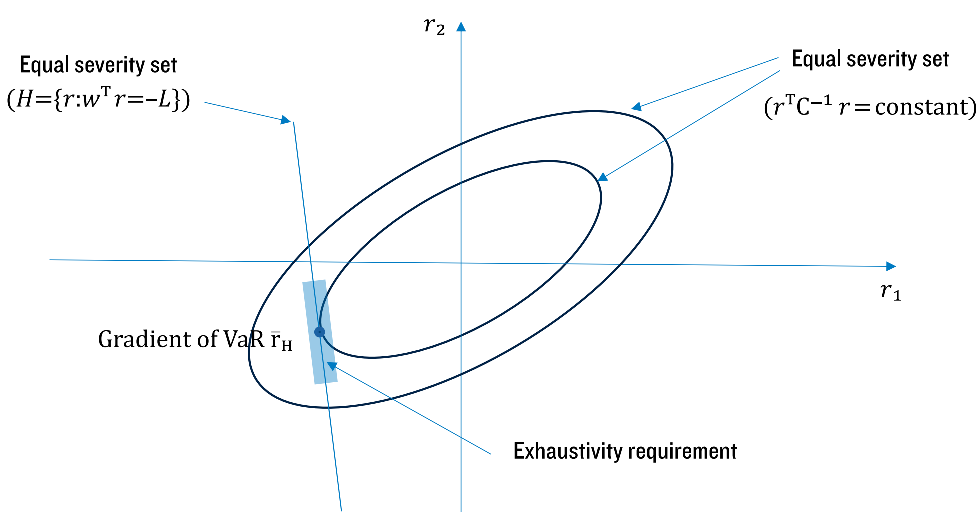
$$\bar{r}_H = q_\alpha \frac{Cw}{\sqrt{w^T C w}}$$

The gradient of VaR is used to compute the risk contributions of positions.

Geometrically, the vector is tangent to the ellipsoid $r^T C^{-1} r = k$ for some constant k.[1] In fact, $\bar{r}_H$ is also the maximum of the conditional distribution density over the hyperplane H, and consequently the most plausible scenario. See Figure 2 for a depiction. But, now we can include more scenarios in the neighborhood of $\bar{r}_H$ that lie on H. Moreover, we can explicitly compute the likelihood of these scenarios since the conditional distribution can be estimated.

From Figure 2, we see that H is a line for two assets and that equally plausible scenarios are pairs of points on this line that intersect iso-probability ellipses. In general, H is a (n-1)-dimensional hyperplane for n assets, and the equal probability scenarios will lie on a (n-1)-dimensional ellipsoid. Exhaustivity is achieved by uniformly selecting a sufficient number of scenarios that lie in the intersection of H and the iso-probability ellipses.

Figure 2: Hyperplane of Portfolio Losses



Source: PGIM Quantitative Solutions. For Illustrative Purposes Only.

[1]Recall that $r^T C^{-1} r$ appears in the multivariate normal density function. Thus, the density function is constant when this term is constant.

Reverse Stress Testing in a High Inflation Environment

In this section we outline how an RST, along with a regular stress test, can be used to analyze portfolios in a higher inflation environment. The rate of inflation in the United States has become an ongoing concern. It has reached 8.5% as of end of March 2022 and continues to increase. In the recent note Portfolio Implications of a Higher US Inflation Regime, the authors discuss the drivers and implications of higher inflation across different asset classes such as equities, bonds, commodities, and real estate. In particular, they discuss the asset class performance in inflationary regimes, and demonstrate that certain real assets such as TIPS, REITs, and commodities have inflation-hedging properties.

We emphasize that RSTs should not be used in isolation but used with other risk analysis including traditional stress testing and risk contribution estimates. As an example, we consider two portfolios listed in Table 2. The first is a traditional 60/40 equity/bond portfolio, while the second portfolio is a blended portfolio consisting of US Equities, US Treasuries, US investment grade bonds, along with the real assets comprised of TIPS, Commodities, REITs, and precious metals.

Recall that the scenarios extracted under RSTs will depend on:

- Portfolio composition
- Distributional assumption of risk factors (used to extract plausibility)

We select the 60/40 portfolio as our base portfolio and specify an input portfolio loss of 10%. In order to extract both scenarios and measure plausibility, we use the conditional covariance matrix from [1], which was derived using a sample from 1973 to 2021 of periods when the year over year US CPI exceeded 4%. Note we can use other covariance matrices under our RST algorithm.

With our inputs specified (portfolio asset weights, portfolio loss, and stressed conditional covariance matrix), we extract the RST scenarios. Table 3 provides sample scenarios that correspond to the base portfolio loss of 10%. The highlighted rows represent the equity and fixed income scenarios. To verify the portfolio loss of 10% from Table 3, we simply apply the asset weights of the 60/40 portfolio to these scenarios. For instance, under scenario 2 we have 0.60(-14.17%) + 0.40(-3.74%) = -10%. Moreover, since the RST scenario generation is a function of not only the portfolio composition but also the underlying model of plausibility (which contains the other asset classes), we are able to extract RST scenarios for the remaining asset class risk factors listed in Table 3.

Note that the base scenario in Table 3 corresponds to the most plausible scenario. The eight nearby scenarios, each generating a portfolio loss of 10%, are 50% as likely as the base scenario.[2] In this table we only provided eight scenarios as an illustration, but under an RST program, we would select many more scenarios and losses in a neighborhood of the 10% loss.

From Table 3, we observe that US Equities have the greatest loss in all scenarios. The losses from all the remaining asset classes are not as severe. In fact, we observe that commodities provide a hedge (positive P&L) in all but one scenario and that precious metals can also provide a hedge. The losses in US Treasuries, US IG Bonds, TIPs, and REITs are not as severe as US Equites, thus also partially mitigating losses.

Combining RST and Traditional Stress Tests

Given these RST scenarios, how does the blended portfolio perform? Recall that the RST scenarios are a function of the portfolio composition (in this case the 60/40 portfolio) and the underlying model of plausibility. Here we apply traditional stress tests to the blended portfolio with the RST scenarios. These scenarios, which were outputs from the RST, are now inputs for the traditional stress tests.

Table 4 provides the P&L contributions for the blended portfolio that correspond to the RST scenarios listed in Table 3. These P&L contributions are computed by applying the blended asset weights listed in Table 2 to the scenarios. As an example, consider scenario 1 in Table 4. The P&L contribution from commodities is 0.08(0.272) = +2.17%. All other P&L contributions are computed in a similar fashion.

From Table 4, we observe that the portfolio loss is less than 10%, highlighting that the addition of real assets provides a mitigating benefit against high inflation regimes.

[2] Relative likelihood of 50% was computed by setting the ratios of the conditional density function

Table 2: Sample Portfolios		
Asset	**Traditional 60/40**	**Blended Portfolio**
US Treasuries	40%	18.8%
US IG Bonds	-	5.0%
US Equities	60%	51.2%
TIPS	-	3.0%
Commodities	-	8.0%
REITS	-	8.0%
Precious Metals	-	6.0%

Sources: Datastream, Bloomberg, FactSet, PGIM Quantitative Solutions. Data as of 12/31/2021. For Illustrative Purposes Only.

Table 3: RST Scenarios for Traditional 60/40 Portfolio Loss of 10%

		Scenario Shifts (%)							
Asset	**Base**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
US Treasuries	-2.83	-2.88	-3.74	-8.04	-4.48	-1.70	-0.96	-3.96	-1.18
US IG Bonds	-6.62	-6.65	-8.26	-13.15	-11.02	-10.34	-7.12	-2.90	-2.22
US Equities	-14.78	-14.75	-14.17	-11.31	-13.68	-15.53	-16.02	-14.03	-15.88
TIPS	-3.56	-1.32	-5.24	-9.74	-5.31	-0.47	-5.15	-6.64	-1.80
Commodities	6.48	27.12	-11.45	17.52	5.64	6.46	6.58	6.51	7.32
REITs	-10.83	-10.34	-7.69	-7.53	-22.25	-10.12	-10.79	-11.55	0.58
Precious Metals	-0.65	33.39	10.28	-7.00	0.13	-0.86	-0.61	-0.46	-1.43
Relative Likelihood	100%	50%	50%	50%	50%	50%	50%	50%	50%

Sources: Datastream, Bloomberg, FactSet, PGIM Quantitative Solutions. Data as of 12/31/2021. For Illustrative Purposes Only.

Table 4: P&L Contributions of Blended Portfolio corresponding to RST Scenarios

		Portfolio P&L Contributions(%)							
Asset	**Base**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
US Treasuries	-0.53	-0.54	-0.70	-1.51	-0.84	-0.32	-0.18	-0.74	-0.22
US IG Bonds	-0.33	-0.33	-0.41	-0.66	-0.55	-0.52	-0.36	-0.15	-0.11
US Equities	-7.57	-7.55	-7.26	-5.79	-7.01	-7.95	-8.20	-7.18	-8.13
TIPS	-0.11	-0.04	-0.16	-0.29	-0.16	-0.01	-0.15	-0.20	-0.05
Commodities	0.52	2.17	-0.92	1.40	0.45	0.52	0.53	0.52	0.59
REITs	-0.87	-0.83	-0.61	-0.60	-1.78	-0.81	-0.86	-0.92	0.05
Precious Metals	-0.04	2.00	0.62	-0.42	0.01	-0.05	-0.04	-0.03	-0.09
Total P&L	-8.92	-5.12	-9.44	-7.87	-9.88	-9.15	-9.27	-8.70	-7.97

Sources: Datastream, Bloomberg, FactSet, PGIM Quantitative Solutions. Data as of 12/31/2021. For Illustrative Purposes Only.

Concluding Remarks

In this note we briefly described reverse stress testing for a specific regime of elevated inflation. Under traditional stress tests, risk factor shocks are explicitly selected while reverse stress tests work in the opposite direction. Under a reverse stress test, we specify the portfolio loss, and then extract scenarios that generate this loss.

Reverse stress testing is a complementary analysis to risk modeling/traditional stress test. Reverse stress testing is attractive because many more scenarios, other than just the most plausible scenario from a classical risk decomposition, can be extracted. This allows managers to examine more scenarios which may reveal hidden or unanticipated risks.

RST requires the modeling of the dependence structure of risk factors, which provides a measure of plausibility. The dependence structure of risk factors can be embedded within a covariance matrix of asset returns. Moreover, utilizing conditional covariance matrices constructed over stressed periods are natural inputs under RST analysis. We provided such an analysis by considering an example involving a traditional 60/40 portfolio and a blended portfolio with real assets under a high inflation regime. Here we incorporated the conditional stressed asset covariance matrix from [1] and found that real assets can help mitigate risk over high inflation environments.

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